Math 3280 Practice Final

This is somewhat longer than the actual exam, which will probably be 5 questions.

(1) Find the general solution to $(1+t)y' + y = \cos t$.

(2) Rewrite the initial value problem $y''' + y'' + y = t$, $y(0) = y'(0) = y''(0) = 0$ as an equivalent first-order system.

(3) Find the general solution to the system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(4) Are the vectors $v_1 = (1, 2, 3, 4)$, $v_2 = (2, -2, 4, 2)$, and $v_3 = (0, -3, -1, -3)$ linearly independent? If not, write one of them as a linear combination of the other two.

(5) Solve the initial value problem $y'' + y = \cos x$, $y'(0) = 0$, $y(0) = -\frac{1}{2}$.

(6) Use Euler's, the Improved Euler's, or the Runge-Kutta method to numerically approximate $y(2)$ to three digits of accuracy if $y' = t + \sqrt{y}$ and $y(0) = 1$.

(7) Find the general solution to the system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(8) Find the Laplace transform $X(s) = \mathcal{L}(x(t))$ if $x'' + 8x' + 15x = 0$ and $x(0) = 0$, $x'(0) = 1$. Then find $x(t)$ using any method.

(9) What is the **form** of the general solution to the ODE $y''' - 4y'' + 14y' - 20y = te^t \cos(3t) + t^2$. Hint: one of the roots of the characteristic polynomial of the left-hand side is 2.