

Math 3280 Worksheet 7

Group members (1 to 3): _____
Equilibria and Runge-Kutta

(1) Sketch the phase plot (equilibria and direction of solutions) for the ODE $y' = y \sin y$.

(2) For the initial value problem $y(1) = 1$ and $xy' = y \frac{y-x}{x+y}$, approximate $y(2)$ by using a 1-step 4th-order Runge-Kutta method (formulae on reverse).

- (3) For extra credit try to find the most accurate answer you can; a bonus will be given for the answer with the most correct digits.

The 4th-order Runge-Kutta formulae are:

$$\begin{aligned}k_1 &= f(x_n, y_n) \\k_2 &= f(x_n + h/2, y_n + hk_1/2) \\k_3 &= f(x_n + h/2, y_n + hk_2/2) \\k_4 &= f(x_n + h, y_n + hk_3)\end{aligned}$$

Finally

$$\begin{aligned}y_{n+1} &= y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6 \\x_{n+1} &= x_n + h\end{aligned}$$