Math 3280 Assignment 4, due Thursday, July 11th.

This assignment covers material from Section 3.7 through 5.3.

(1) Find a quadratic polynomial $a_2x^2 + a_1x + a_0$ whose graph passes through the points $(1, 3)$, $(2, 3)$, and $(4, 9)$.

(2) Determine whether the vectors $(0, 2)$ and $(0, 5)$ are linearly dependent or independent.

(3) Express $w = (1, 2)$ as a linear combination of $u = (-1, -1)$ and $v = (2, 1)$.

(4) Calculate the determinate of the matrix whose columns are u, v, and w to determine if u, v, and w are linearly independent or not, with $u = (-2, -5, -4)$, $v = (5, 4, -6)$, and $w = (8, 3, -4)$.

(5) Is the subset $W = \{(x, y, z) \mid y \geq 0\} \subset \mathbb{R}^3$ a vector subspace of $\mathbb{R}^3$? Explain why or why not.

(6) Is the subset $W = \{(x, y, z) \mid y = 0\} \subset \mathbb{R}^3$ a vector subspace of $\mathbb{R}^3$? Explain why or why not.

(7) Is the subset $W = \{(x, y, z) \mid z = 1\} \subset \mathbb{R}^3$ a vector subspace of $\mathbb{R}^3$? Explain why or why not.

(8) If W is the subset of all vectors (x, y) in $\mathbb{R}^2$ such that $|x| = |y|$, is W a vector subspace or not?

(9) Suppose that x_0 is a solution to the equation $Ax = b$ (where A is a matrix and x and b are vectors). Show that x is a solution to $Ax = b$ if and only if $y = x - x_0$ is a solution to the system $Ay = 0$.

(10) Determine whether the vectors $v_1 = (3, 0, 1, 2)$, $v_2 = (1, -1, 0, 1)$, and $v_3 = (4, 2, 2, 2)$ are linearly independent or dependent. If they are linearly dependent, find a non-trivial combination of them that adds up to the zero vector.

(11) Find a basis for the subspace of $\mathbb{R}^3$ given by $x - 2y + 7z = 0$.

(12) Find a basis for the subspace of $\mathbb{R}^3$ given by $x = z$.

(13) Find a basis for the subspace of all vectors (x_1, x_2, x_3, x_4) in $\mathbb{R}^4$ such that $x_1 + x_2 = x_3 + x_4$.

The following two questions are about subsets of the set of real-valued functions of the real line. We will call this set $\mathcal{F}$.

(14) Is the subset of $\mathcal{F}$ with the property that $f(0) = 0$ a vector space?

(15) Is the subset of $\mathcal{F}$ with the property that $f(-x) = -f(x)$ for all x a vector space?

(16) Compute the Wronskian of $f_1 = e^{-x}$, $f_2 = \cos(x)$ and $f_3 = \sin(x)$ to determine whether these three functions are linearly independent on the real line.

(17) Solve the initial value problem $y'' - 4y = 0$, $y(0) = 4$, $y'(0) = 2$ given that $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ are both solutions to the ODE.

(18) Find the general solution to $y'' + 6y' = 0$.

(19) Find the general solution to $4y'' + 4y' + y = 0$.

(20) For what second-order constant coefficient linear homogeneous ODE would $y = C_1 + C_2x$ be the general solution?

(21) Show that the functions $3x$, $2x^2$, and $5x - 8x^2$ are linearly dependent by finding a linear combination of them that equals zero.

(22) Find the general solution to $y'' + 10y' + 25y = 0$.

(23) Find the general solution to $y^{(4)} - 6y^{(3)} + 9y'' = 0$.