Math 3280 Assignment 5, due Thursday, July 18th.

For this assignment you should read chapter 5 in the text.

(1) Solve the initial value problem $y'' - 6y' + 25y = 0$, $y(0) = 6$, $y'(0) = 2$.

(2) Find the general solution of $6y^{(4)} + 5y^{(3)} + 18y'' + 20y' - 24y = 0$ given that $y = \cos(2x)$ is a solution.

(3) Consider the differential equation $y'' + sgn(x)y = 0$, where $sgn(x)$ is the sign function:

$$sgn(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Compute the two linearly independent solutions y_1 and y_2 of this differential equation which satisfy the initial conditions $y_1(0) = 1$, $y_1'(0) = 0$ and $y_2(0) = 0$, $y_2'(0) = 1$.

(4) Find a particular solution to the ODE $y'' - y' + 2y = 4x + 12$.

(5) Find a particular solution to the ODE $y'' - y' + y = \sin^2(x)$. (Hint: it may be helpful to use a trig identity.)

(6) Find the general solution to $y^{(3)} - y' = e^x$.

For the following two problems, determine the form of the particular solution - note that **you do not have to determine the values of the coefficients**. You should not include terms from the homogeneous (complementary) solution.

(7) Determine the form of the particular solution to $y''' = 9x^2 + 1$.

(8) Determine the form of the particular solution to $y^{(4)} - 16y'' = x^2\sin(4x) + \sin(4x)$.

(9) Solve the initial value problem $y'' + 2y' + 2y = \cos(3x)$, $y(0) = 0$, $y'(0) = 2$.

(10) Solve the initial value problem $y^{(4)} - y = 1$, $y(0) = y'(0) = y''(0) = y^{(3)} = 0$.

(11) Use the variation of parameters method to find the general solution of

$$y'' - 2y' + y = e^x/x$$

(12) Use the variation of parameters method to find the general solution of

$$y'' + 9y = 12\sec(3x).$$

(13) How many times can an overdamped mass-spring system ($mx'' + cx' + kx = 0$ with $c^2 > 4mk$; c, m, and k are non-negative) with arbitrary initial conditions $x(0) = x_0$, $x'(0) = v_0$ pass through $x = 0$? What if it is critically damped ($c^2 = 4mk$)?

(14) Find the steady-state solution of the forced, damped oscillator $x'' + x'/4 + 2x = 2\cos(wt)$ if $x(0) = 0$ and $x'(0) = 4$. Sketch the overall amplitude of the steady-state solution as a function of w.

(15) Rewrite the second-order differential equation $x'' + 3x' + 5x = t$ as a system of first-order differential equations. (You do not have to find the solution.)