Math 3298 Practice Final
This is roughly 50% longer than the actual exam.

(1) Reverse the order of integration for the integral
\[ \int_0^1 \int_x^1 \int_0^y f(x, y, z) \, dz \, dy \, dx. \]

(2) Compute the vector line integral
\[ \int_C \vec{F} \cdot d\vec{r} \] where \( C \) is the path \((\pi t, \pi t^2), t \in [0, 1]\), and \( \vec{F} = (1 + y \sin(x) + \sin(y), x \cos(y) - \cos(x)) \).

(3) Find the linearization of \( f(x, y) \) at \((x, y) = (0, 1)\) if \( f = h(u(x, y), v(x, y)) \) and \( \nabla h\big|_{(1, 1)} = (\frac{\partial h}{\partial u}, \frac{\partial h}{\partial v})\big|_{(1, 1)} = (2, 3) \), \( u(x, y) = x + y \), and \( v(x, y) = y^2 \).

(4) Find the surface area of the torus parameterized by \( x = (2 + \cos(v)) \cos(u) \), \( y = (2 + \cos(v)) \sin(u) \), \( z = \sin(v) \), with \( u \in [0, 2\pi] \) and \( v \in [0, 2\pi] \).

(5) Find the maxima and minima of \( f(x, y) = \frac{1}{x} + \frac{2}{y} \) on the set \( \frac{1}{x^2} + \frac{1}{y^2} = 1 \).

(6) Find the volume of the solid wedge bounded by the planes \( z = 0 \) and \( z = -y \) and the cylinder \( x^2 + y^2 = 4 \) (with \( y \geq 0 \)).

(7) Use Green’s Theorem to find the smooth, simple, closed and positively oriented curve in the plane for which the line integral \( \oint_C (x^2 y^4 + y^3) \, dx + x \, dy \) has the largest possible value.

(8) Compute the value of \( \int_S (\nabla \times \vec{F}) \cdot \vec{n} \, dS \) where \( S \) is the upper half of the ellipsoid \( 4x^2 + 9y^2 + 36z^2 = 36 \), \( z \geq 0 \), with upward pointing normal, and \( \vec{F} = (y, x^2, (x^2 + y^2)^{3/2} e^{xyz}) \).

(9) Let \( \vec{r}(t) \) be a curve in space with unit tangent, normal, and binormal vectors \( \vec{T}, \vec{N}, \) and \( \vec{B} \). Show that \( \frac{d\vec{B}}{dt} \) is perpendicular to \( \vec{T} \).