Assignment 10, due Wednesday, April 20th

(1) Find the (implicit) equation for the tangent plane to the surface $\vec{r} = (u^2, v^2, uv)$ at the point $(1, 1, 1)$.

(2) Compute the surface flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$ and $\vec{F} = (x, 2y, 3z)$.

(3) Compute the surface flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the portion of the cylinder $x^2 + y^2 = 1$ between the planes $z = 0$ and $z = 1$, with outward pointing normal, and $\vec{F} = (x, y, z)$.

(4) Use Stokes' Theorem to compute $\int\int_S \nabla \times \vec{F} \cdot d\vec{S}$ where $\vec{F} = (2y\cos(z), e^x \sin(z), xe^y)$ and S is the upper hemisphere of $x^2 + y^2 + z^2 = 9$ with upward-pointing normal (i.e the z-component is positive).

(5) Suppose that C is a simple closed curve in the plane $x + y + z = 1$, positively oriented with respect to the normal vector $(1, 1, 1)$. Compute the line integral $\oint_C (z, 2x, -y) \cdot d\vec{r}$ in terms of the area enclosed by C.

(6) Use Stokes' Theorem to compute $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 y, x^3/3, xy)$ and C is the intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$. C is oriented counter-clockwise when viewed from above.

(7) Use the divergence theorem to write the volume of a three-dimensional region R as a surface flux integral.

(8) Determine the sign of the divergence of the vector field in the figure below at the indicated points P_1 and P_2.