Homework 2, due Wednesday, January 27th in class.

(1) Determine the type of quadric surface defined by $x^2 + (\frac{y}{9})^2 + z^2 = 1$ and describe its intersection with the plane $y = 0$.

(2) Describe the intersection of a plane $z = s$ with the surface given by $x^2 + 4y^2 - 4z^2 = -1$. For which values of s is the intersection empty?

(3) Rewrite the quadric surface $z = x^2 - y^2$ in spherical coordinates in the form $\rho = f(\theta, \phi)$.

(4) Match the following 3D parametric curves to the six images shown on the following page.

(a) $x = \cos(10t)$, $y = t$, $z = \sin(10t)$.

(b) $x = t$, $y = t^2$, $z = e^{-t}$.

(c) $x = t$, $y = 1/(1+t^2)$, $z = t^2$.

(d) $x = e^{-t} cos(10t)$, $y = e^{-t}\sin(10t)$, $z = e^{-t}$.

(e) $x = \cos(t)$, $y = \sin(t)$, $z = \sin(5t)$.

(f) $x = \cos(t)$, $y = \sin(t)$, $z = \ln(t)$.

(5) Find $\vec{r}'(t)$ and sketch the plane curve $\vec{r}(t) = (1+t, \sqrt{t})$. Include the vectors $\vec{r}(1)$ and $\vec{r}'(1)$ in your sketch.

(6) Find the unit tangent vector $\vec{T}(t)$ of the curve $\vec{r} = 4\sqrt{t}\,\vec{i} + t^2\,\vec{j} + t\,\vec{k}$ at $t = 1$.

(7) Find parametric equations for the tangent line to $x = t^2 - 1$, $y = t^2 + 1$, $z = t + 1$ at the point $(-1, 1, 1)$.

(8) If $u(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}''(t)]$, show that $u'(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$.

(9) Find the length of the curve $\vec{r}(t) = (3\cos 2t, 3\sin 2t, 3t)$ with t in $[0, \pi/2]$.

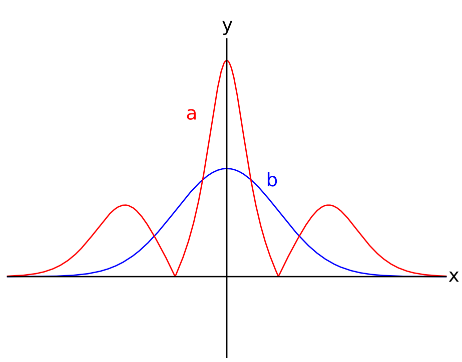
(10) Find the length of the curve $\vec{r}(t) = (2\cos 3t, 2\sin 3t, 2t^{3/2})$ with t in $[0, 1]$.

(11) Parameterize the curve $\vec{r}(t) = (2\cos 3t, 2\sin 3t, 2t^{3/2})$ by arc length.

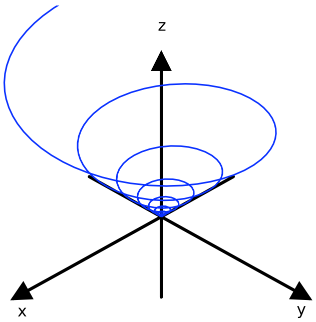
(12) Find the unit tangent $\vec{T}$, unit normal $\vec{N}$, and curvature κ of the curve $\vec{r}(t) = (t^2, 2t, \ln(t))$ when $t = 4$.

(13) For what value of x is the curvature of the curve $y = e^x$ maximized? What is the limit of the curvature as $x \to \infty$?

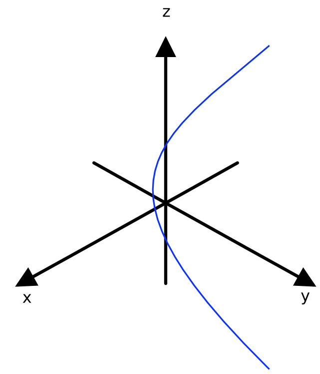
(14) Two graphs are shown below; one is a curve $y = f(x)$ and the other is the curvature $\kappa(x)$ of that curve. Identify which is which.



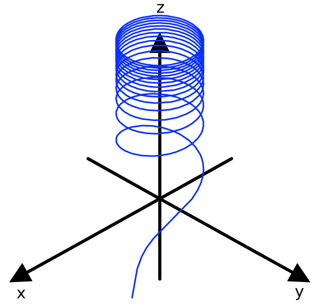
I



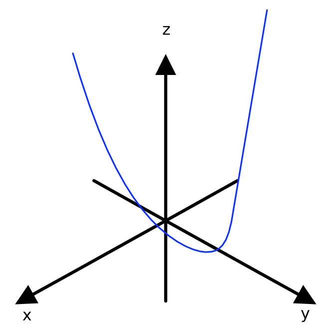
II



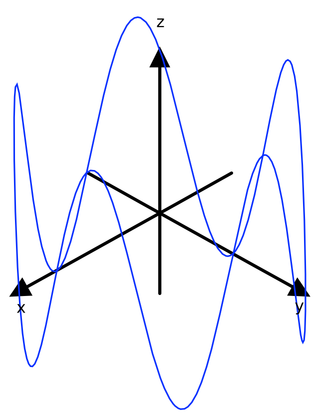
III



IV



V



VI