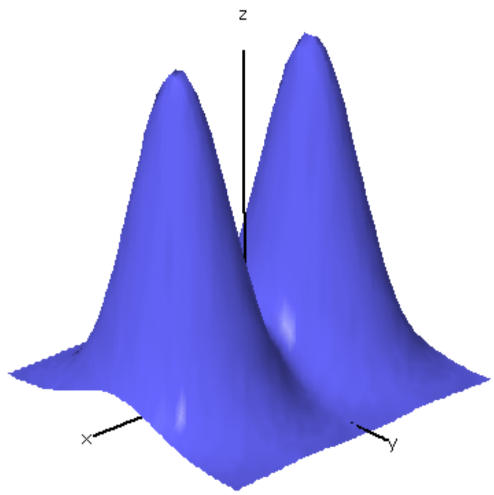
# Homework 3, due Wednesday February 3rd.

This assignment is on material covered in sections 14.1 to 14.4 in Stewart's text. Reading (or at least skimming) those sections is highly recommended.

(1) Sketch the contour map of the function whose graph is shown below.




(2) Sketch a contour map of the function $f = x^2 - y^2$.

(3) Sketch a contour map of the function $f = e^{y/x}$.

(4) Compute the following limits:

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$ (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^4 \sin(xy)}{x^2 + y^2}$

For Exercises 5 - 8, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(5) $f(x, y) = e^{xy} + xy$

(6) $f(x, y) = x^4$

(7) $f(x, y) = \dfrac{x+1}{y+1}$

(8) $f(x, y) = \ln(xy) + x^y$

For Exercises 9 and 10, calculate all four second-order partial derivatives of the function and verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

(9) $f(x, y) = x^2 + y^2$

(10) $f(x, y) = \cos(x + y)$

(11) The ideal gas law is $PV = cT$ where $P$ is the pressure, $V$ is the volume, $T$ is the temperature and $c$ is a constant. Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

(Treat each variable as a function of the other two.)

(12) Find the linearization $L(x, y)$ of the function $f(x, y) = x\sqrt{y}$ at the point $(1, 4)$.