

HOMEWORK 4, DUE FEBRUARY 10TH

- (1) Find the equation of the plane that is tangent to the surface $z = y \ln(x)$ at the point $(1, 4, 0)$.
- (2) Use the linearization of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ to approximate $f(1.95, 1.05)$.
- (3) Use the chain rule to compute $\frac{dz}{dt}$ if $z = x^2y + xy^2$, $x = 2 + t^4$, $y = 1 - t^3$.
- (4) Use the chain rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for $z = x^2 + xy + y^2$, $x = s + t$, $y = st$.
- (5) Use the chain rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for $z = x/y$, $x = se^t$, $y = 1 + se^{-t}$.
- (6) Suppose that $f(x, y)$ is a differentiable function, and $g(r, s) = f(2r - s, s^2 - 4r)$. Compute $\frac{\partial g}{\partial r}|_{(r=1, s=2)}$ and $\frac{\partial g}{\partial s}|_{(r=1, s=2)}$ from the table of values below:

| | f | g | $\partial f/\partial x$ | $\partial f/\partial y$ |
|----------|-----|-----|-------------------------|-------------------------|
| $(0, 0)$ | 3 | 6 | 4 | 8 |
| $(1, 2)$ | 6 | 3 | 2 | 5 |

- (7) Compute the gradient of the function $f(x, y) = x^2e^y$.
- (8) Compute the value of the gradient of the function $f(x, y, z) = \sin(xyz)$ at the point $(1, 1, \pi/4)$.
- (9) Compute the directional derivative of $f(x, y) = x^2 + y^2 + 1$ at the point $(1, 1)$ in the direction $\vec{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
- (10) Compute the directional derivative of $f(x, y) = x^2y^3 - y^4$ at the point $(2, 1)$ in the direction of the ray $\theta = \pi/4$.
- (11) Compute the directional derivative of $f(x, y, z) = x^2e^{yz}$ at the point $(1, 1, 1)$ in the direction of $\vec{u} = (1, 1, 1)$.
- (12) Find all of the directions in which the directional derivative of $f(x, y) = \sin(xy) + x^2$ at the point $(1, 0)$ has the value 1.
- (13) Find the local maxima and minima of $f(x, y) = x^3y + 12x^2 - 8y$.