Homework 7, Due Wednesday, March 16th

(1) Compute the integral $\displaystyle\int\int_R (x+y)\,dA$ where R is the region bounded by $y=\sqrt{x}$ and $y=x^2$.

Reverse the order of integration of the following two double integrals after sketching the region of integration. Note that you are **not** required to evaluate the resulting integral.

(2) $\displaystyle\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} f(x,y)\,dx\,dy$ (3) $\displaystyle\int_0^1 \int_{2x}^{2} f(x,y)\,dy\,dx$

(4) Sketch the region of integration of the polar integral $\displaystyle\int_0^{\pi/2} \int_0^{4\cos(\theta)} r\;dr\;d\theta$, and then compute its value.

(5) Compute the volume of the solid bounded by the paraboloid $z=10-3x^2-3y^2$ and the plane $z=4$ by using polar coordinates.

(6) Compute the following sum of integrals by sketching the total region of integration and then switching to polar coordinates:

$$\int_{1/\sqrt{2}}^{1} \int_{\sqrt{1-x^2}}^{x} xy\;dy\;dx + \int_1^{\sqrt{2}} \int_0^x xy\;dy\;dx + \int_{\sqrt{2}}^{2} \int_0^{\sqrt{4-x^2}} xy\;dy\;dx.$$

(7) Find the center of mass of a lamina with density $\rho = cxy$, if the lamina is the rectangle $[0,a]\times[0,b]$, where a, b, and c are all positive constants.

(8) Consider a lamina which is a quarter of the unit disk, that is $x^2+y^2\le 1$ with $x\ge 0$, $y\ge 0$, and whose density is proportional to the distance from the y-axis. Find the center of mass.

(9) Find the moments of inertia (I_x, I_y, and I_0) of a lamina with density $\rho(x,y)=y$ on the region bounded by $y=e^x$, $y=0$, $x=0$, and $x=1$.

(10) Compute the value of the triple integral $\displaystyle\int_0^1 \int_0^y \int_0^{x+y} 12xy\;dz\;dx\;dy$.

(11) Compute the value of the triple integral $\displaystyle\int_0^2 \int_y^{2y} \int_0^x 2xyz\;dz\;dx\;dy$.

(12) Compute the integral $\displaystyle\int\int\int_\Omega x\;dV$ where Ω is the region $x\le 4$, $x\ge 4y^2+4z^2$.

(13) Find the center of mass of the solid bounded by the planes $x=0$, $z=0$, $x+z=1$, and the parabolic cylinder $z=1-y^2$ with density $\rho(x,y,z)=4$. A picture of the boundaries of this solid is shown below.