Homework 8, due Friday March 25

(1) Write down explicit triple integrals for the mass and center of mass of the hemisphere $x^2+y^2+z^2 \le 1$, $z \ge 0$, with density $\rho = \sqrt{x^2+y^2+z^2}$. You do not have to evaluate these integrals (!).

(2) Compute the integral $\int\int\int_R e^{x^2+y^2+z^2}\ dV$ where R is the region inside the sphere $x^2+y^2+z^2=9$ and within the first octant (i.e. $x \ge 0$, $y \ge 0$, $z \ge 0$). A numerical answer is acceptable.

(3) Find the volume of a wedge cut from a ball of radius R by two planes which intersect on a diameter of the ball at an angle of $\pi/6$. (This volume is like a section of an orange.)

(4) Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation $x=(u^2-v^2)/2$, $y=(u^2+v^2)/2$.

(5) Use the transformation $x=\sqrt{2}u-\sqrt{2/3}v$, $y=\sqrt{2}u+\sqrt{2/3}v$ to compute the integral $\int\int_R (x^2-xy+y^2)^{1/2}\ dA$ where R is the region bounded by the ellipse $x^2-xy+y^2=2$.

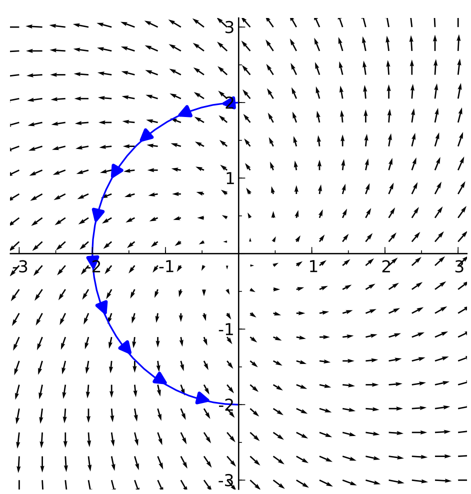
(6) Match the vector field plots on the second page to the following vector fields (as usual the x-axis is the horizontal axis in these plots):
   (I) $\vec{F}=(x-1,x+3)$
   (II) $\vec{F}=(y,x)$
   (III) $\vec{F}=(\sin(x),1)$
   (IV) $\vec{F}=(y,1/x)$

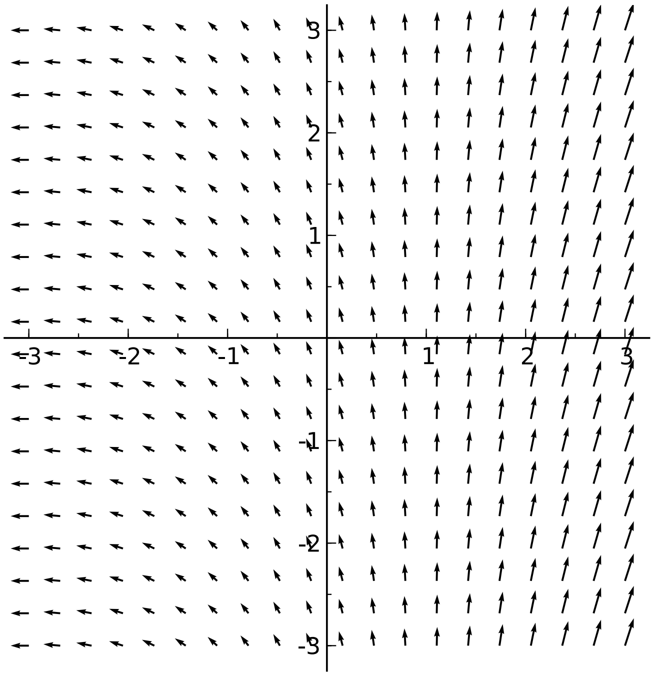
(7) Compute the scalar line integral $\int_C y\ ds$ where C is the curve $x=y^2$ for $y \in [0,3]$.

(8) Compute the scalar line integral $\int_C x/y\ ds$ where C is the curve $x=t^3$, $y=t^4$, $t \in [1/2,1]$.

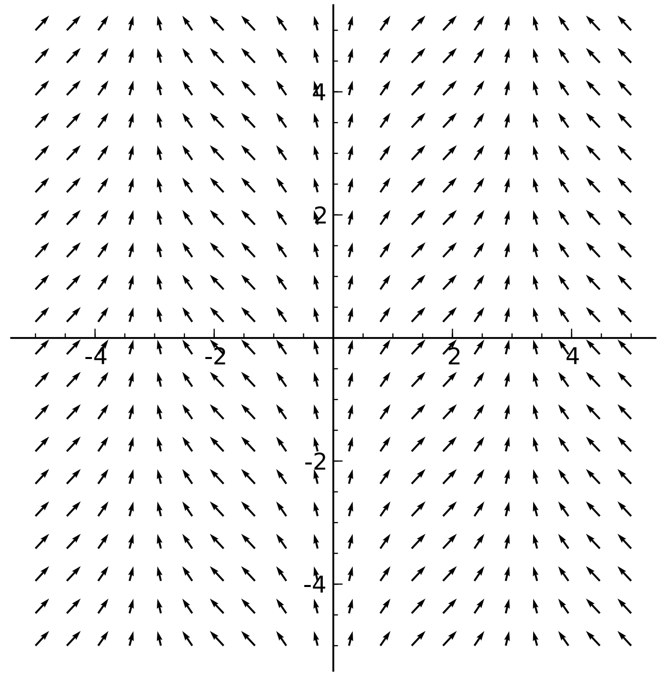
(9) Compute the scalar line integral $\int_C xy^3\ ds$ where C is the curve $x=3\sin(t)$, $y=3\cos(t)$, $z=4t$ for $t \in [0,\pi/2]$.

(10) Is the vector line integral $\int_C \vec{F}\cdot\ d\vec{s}$ positive, negative, or zero for the $\vec{F}$ and C shown below? (The curve C is in blue.)

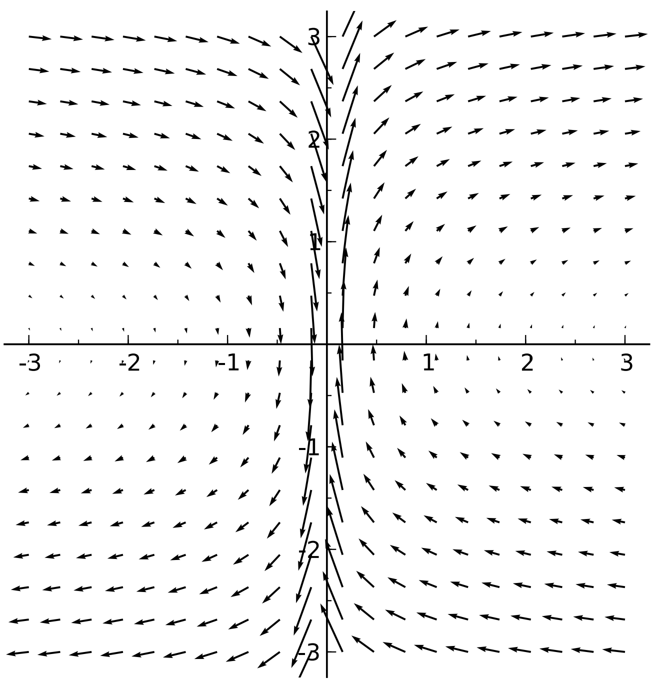




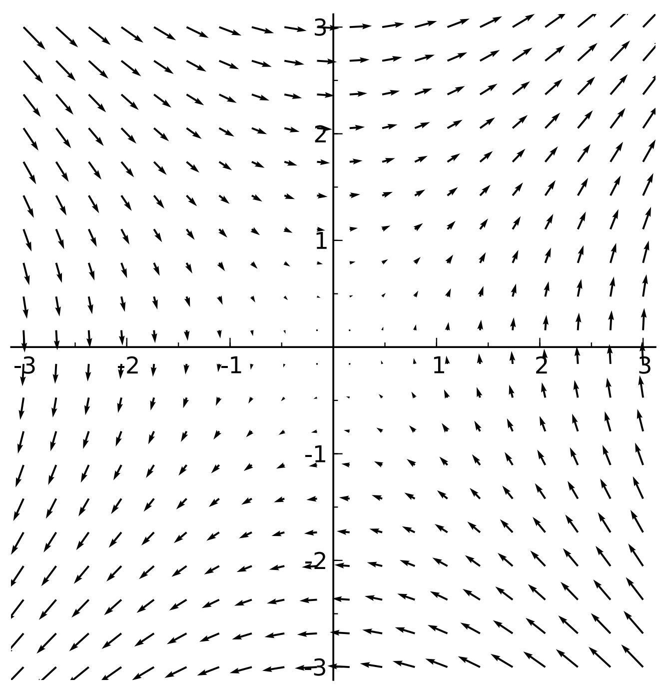
A.



B.



C.



D.