Homework 9, Due Wednesday April 13th.

(1) Compute the vector line integral $\int_C \vec{F} \cdot \, d\vec{s}$ where $\vec{F} = (yz, xz, xy)$ and C is the curve $\vec{r}(t) = (t, t^2, t^3)$ for t in $[0, 1]$.

(2) Find a function f such that $\nabla f = \vec{F}$ where $\vec{F} = (4xy + \ln(x), 2x^2)$. Use this evaluate $\int_C \vec{F} \cdot \, d\vec{s}$ where C is a curve beginning at $(1, 0)$ and ending at $(2, 2)$ along which $x > 0$.

(3) Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (\sqrt{x} + y^3, x^2 + \sqrt{y})$ and C is the graph of $y = \sin(x)$ from $(0, 0)$ to $(\pi, 0)$ and the line segment from $(\pi, 0)$ to $(0, 0)$.

(4) Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ directly and with Green's Theorem if $\vec{F} = (xy^2, x^3)$, and C is the rectangle (oriented positively) $(0, 0)$, $(2, 0)$, $(2, 3)$, and $(0, 3)$.

(5) Calculate the curl and divergence of the vector field $\vec{F} = (2, x + yz, xy - \sqrt{z})$.

(6) Calculate the curl and divergence of the vector field $\vec{F} = (\cos(yz), 0, -\sin(xy))$.

(7) Construct a potential function for the vector field $(e^z, 6, xe^z)$.

(8) Is there a vector field $\vec{G}$ such that $\nabla \times \vec{G} = (xy^2, yz^2, zx^2)$? Explain why or why not.

(9) Prove that $\nabla \cdot (f\vec{F}) = f\nabla \cdot \vec{F} + \vec{F} \cdot \nabla f$ if f is a scalar function of (x, y, z) with continuous partial derivatives and $\vec{F}$ is a vector field whose components also have continuous partial derivatives.

(10) If $\vec{r} = (x, y, z)$, $r = |\vec{r}|$, and $\vec{F} = \vec{r}/r^p$ (p is a real number), compute $\nabla \cdot \vec{F}$. Is there a value of p for which $\nabla \cdot \vec{F} = 0$?

(11) Parameterize the plane which contains the point $(1, 1, 1)$ and which is parallel to the vectors $(1, 0, -1)$ and $(1, -1, 0)$.

(12) Parameterize the lower half of the ellipsoid $x^2/9 + y^2/4 + z^2 = 1$.

(13) Compute the surface area of the part of the plane $3x + 2y + z = 6$ that is inside the cylinder $x^2 + y^2 = 16$.