Math 3298 Practice Midterm 1

This practice test is over twice as long as the actual exam. The material covered is in chapters 13, 14, and up to chapter 15.5 in the Stewart text. The emphasis will be on multivariable differential calculus (chapter 14).

(1) (a) Use the formula $\kappa = \dfrac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$ to show that for a parameterized plane curve $(x(t), y(t))$ the curvature is

$$\kappa = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{|\dot{x}^2 + \dot{y}^2|^{3/2}}$$

where a dot denotes a derviative with respect to time.

(b) Use the result of part (a) to compute the curvature of $x(t) = 1 + t^3$, $y(t) = t + t^2$.

(2) Classify the critical points of $f(x, y) = 2y^2 + 2xy - y - x^3 + x + 1$.

(3) Compute the limit $\dfrac{x^2 + y\sin(y)}{x^2 + y^2}$ if it exists, or show why it does not exist.

(4) Find the curvature of $\vec{r}(t) = (t^2, t^3, 2t^3)$ at $t = 1$.

(5) Use the linearization of the function $f(x, y) = x + \ln(xy)$ at $(x, y) = (2, 1/2)$ to find an approximate value for $f(1.9, .4)$.

(6) Find three positive numbers x, y, and z such that $x + 2y + 3z = 7$ and for which the function $f(x, y, z) = x^2y^2z^3$ is maximized.

(7) Use the chain rule to compute $\frac{\partial z}{\partial t}$ at $t = 2$ if $z = \sin(xy)\sin(y)$ and $x = 1/t$, $y = f(t)$ where $f'(2) = 3$ and $f(2) = \pi$.

(8) Find the directions in which the directional derivative of $f(x, y) = x^2 + 2y^2 - 4y$ at the point $(1, 1)$ has the value 1.

(9) Find the integral of the function $f(x, y) = 2x\sqrt{y^2 - x^2}$ over the triangle $T = \{(x, y) \mid 0 \leq y \leq 2, 0 \leq x \leq y\}$