Math 3298 Practice Midterm 2 problems
The actual exam will consist of three or four required questions and an optional extra credit question.
(1) Find the average value of the function $1+3 x+y$ on the triangle with vertices $(0,0),(1,0)$, and $(0,2)$.
(2) Find the volume of the solid inside the sphere $x^{2}+y^{2}+z^{2}=9$ and outside the cylinder $x^{2}+y^{2}=1$.
(3) Compute the integral $\iiint_{R} \sqrt{x^{2}+y^{2}} d V$ where $R$ is the region inside the cylinder $x^{2}+y^{2}=25$ and between $z=-1$ and $z=4$.
(4) Find the volume of the solid bounded by the planes $z=x, y=x, x+y=2$, and $z=0$.
(5) Change the order of integration of $\int_{0}^{2} \int_{\operatorname{Arctan}(x)}^{\operatorname{Arctan}(\pi x)} d y d x$ and evaluate the integral.
(6) Compute the integral $\int_{-1}^{1} \int_{-\sqrt{1-x^{2}}}^{\sqrt{1-x^{2}}} \int_{x^{2}+y^{2}}^{2-x^{2}-y^{2}}\left(x^{2}+y^{2}\right)^{3 / 2} d z d y d x$ by changing to cylindrical coordinates.
(7) This difficult a problem would be extra credit: Assuming that $\beta \in(0, \pi / 2)$ and $a>0$, compute the following integral

$$
\int_{0}^{a \sin \beta} \int_{y \cot \beta}^{\sqrt{a^{2}-y^{2}}} \int_{0}^{1} \ln \left(x^{2}+y^{2}\right) d z d x d y
$$

