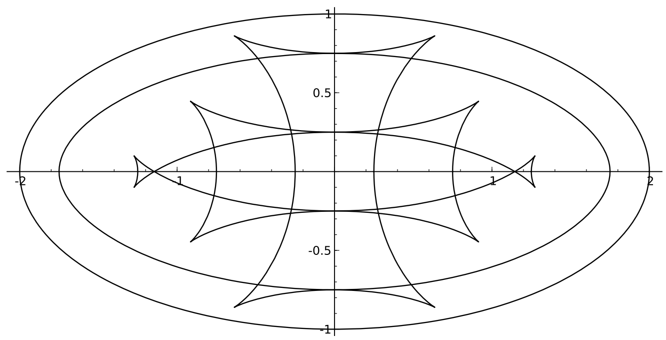
Math 3298 Worksheet 19: review

(1) Find and classify the critical points of $f(x, y) = e^{4y-x^2-y^2}$.

(2) Suppose we have a plane curve $\vec{r}(t) = (x(t), y(t))$, and we construct a second curve $\vec{p}(t)$ parallel to it, a distance c in the direction of the unit normal vector $\vec{N}$ (so $\vec{p} = \vec{r} + c\vec{N}$). An example of such curves are shown in the figure below for the ellipse $\vec{r} = (2\cos(t), \sin(t))$



Find a formula for the curvature of parallel curves in terms of c and the curvature κ of the original curve $\vec{r}$. One helpful trick is that you can assume that $\vec{r}$ is parameterized with respect to arclength, so $|\vec{r}\,'| = 1$.

(3) Minimize the distance from the origin to the surface given by $z^2 = xy + 1$.

(4) Suppose you are feeling lucky and decide to take a hike on the active volcano Mount St. Helens. You take the path indicated by the heavy ellipse on the contour (topographic) map below.

Estimate the location of your highest and lowest point, and sketch the gradient of the height function (the function whose contours are drawn; height is in meters) at those points.

Then pick a third point and draw the gradient of the height function there as well.