

Math 3298 Worksheet 40:

Group members (2 to 4): _____

(1) Suppose the magnitude of a 3D vector field $F = (f_1, f_2, f_3)$ is bounded in the sense that at each point in space $|F| = \sqrt{f_1^2 + f_2^2 + f_3^2} \leq 1$. Use the divergence theorem to find an upper bound on the magnitude of $\iiint_R \operatorname{div} F \, dV$ where R is a region in space with a smooth boundary S .

(2) For the vector field $G = (-2xz, 0, y^2)$:

(a) Compute the curl of G .

(b) Show that $\oint_C G \cdot dr = 0$ for any simple closed smooth curve on the sphere $x^2 + y^2 + z^2 = 4$.