Math 3298 Practice Final

This is roughly twice as long as the actual exam.

(1) Reverse the order of integration for the integral $\int_0^1 \int_x^1 \int_0^{y^2} f(x,y,z)dzdydx$.

(2) Compute the vector line integral $\int_C \vec{F}\cdot d\vec{r}$ where C is the path $(4-3^t, -2+2t, \pi t)$, $t \in [0,1]$, and $\vec{F} = (2x\cos z - x^2, z - 2y, y - x^2 \sin z)$.

(3) Find the linearization of $f(x,y)$ at $(x,y) = (0,1)$ if $f = h(u(x,y), v(x,y))$ and $\text{grad}(h)|_{(1,1)} = (\frac{\partial h}{\partial u}, \frac{\partial h}{\partial v})|_{(1,1)} = (2,3)$, $u(x,y) = x + y$, and $v(x,y) = y^2$.

(4) Find the surface area of the torus parameterized by $x = (2 + \cos(v))\cos(u)$, $y = (2 + \cos(v))\sin(u)$, $z = \sin(v)$, with $u \in [0, 2\pi]$ and $v \in [0, 2\pi]$.

(5) Find the maxima and minima of $f(x,y) = \frac{1}{x} + \frac{2}{y}$ on the set $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

(6) Find the volume of the solid wedge bounded by the planes $z = 0$ and $z = -2y$ and the cylinder $x^2 + y^2 = 4$ (with $y \geq 0$).

(7) Use Green's Theorem to find the smooth, simple, closed and positively oriented curve in the plane for which the line integral $\oint (\frac{x^2 y}{4} + \frac{y^3}{3})dx + xdy$ has the largest possible value.

(8) Compute the value of $\int\int_S (\bigtriangledown \times \vec{F}) \cdot \vec{n}\ dS$ where S is the upper half of the ellipsoid $4x^2 + 9y^2 + 36z^2 = 36$, $z \geq 0$, with upward pointing normal, and $\vec{F} = (y, x^2, (x^2+y^2)^{3/2}e^{xyz})$.

(9) Let $\vec{r}(t)$ be a curve in space with unit tangent, normal, and binormal vectors $\vec{T}$, $\vec{N}$, and $\vec{B}$. Show that $\frac{d\vec{B}}{dt}$ is perpendicular to $\vec{T}$.

(10) Compute the flux integral $\int\int_S \vec{F}\cdot\vec{n}\ dS$ where S is the graph of $z = 1 - x^2 - y^2$, with upward normal, for $z \geq 0$, and with $\vec{F} = (xz, yz, 2z^2)$.