Homework 2, due Friday, May 30th.

(1) Match the following 3D parametric curves to the six images shown on the following page.

(a) $x = \cos(10t), \qquad y = t, \qquad z = \sin(10t).$

(b) $x = t, \qquad y = t^2, \qquad z = e^{-t}.$

(c) $x = t, \qquad y = 1/(1+t^2), \qquad z = t^2.$

(d) $x = e^{-t} cos(10t), \qquad y = e^{-t}\sin(10t), \qquad z = e^{-t}.$

(e) $x = \cos(t), \qquad y = \sin(t), \qquad z = \sin(5t).$

(f) $x = \cos(t), \qquad y = \sin(t), \qquad z = \ln(t).$

(2) Find $\vec{r}'(t)$ and sketch the plane curve $\vec{r}(t) = (1+t, \sqrt{t})$. Include the vectors $\vec{r}(1)$ and $\vec{r}'(1)$ in your sketch.

(3) Find the unit tangent vector $\vec{T}(t)$ of the curve $\vec{r} = 4\sqrt{t}\,\vec{i} + t^2\,\vec{j} + t\,\vec{k}$ at $t = 1$.

(4) Find parametric equations for the tangent line to $x = t^2 - 1$, $y = t^2 + 1$, $z = t + 1$ at the point $(-1, 1, 1)$.

(5) If $u(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}''(t)]$, show that $u'(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$.

(6) Find the length of the curve $\vec{r}(t) = (3\cos 2t, 3\sin 2t, 3t)$ with t in $[0, \pi/2]$.

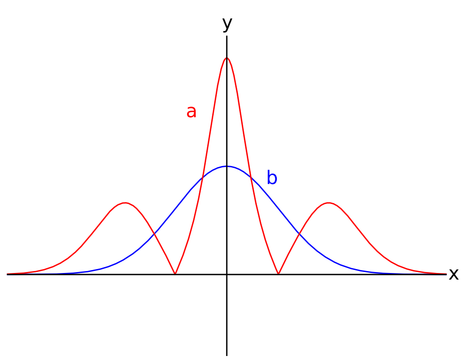
(7) Find the length of the curve $\vec{r}(t) = (2\cos 3t, 2\sin 3t, 2t^{3/2})$ with t in $[0, 1]$.

(8) Parameterize the curve $\vec{r}(t) = (2\cos 3t, 2\sin 3t, 2t^{3/2})$ by arc length.

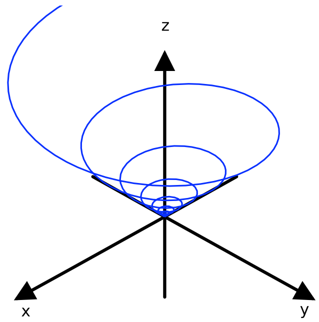
(9) Find the unit tangent $\vec{T}$, unit normal $\vec{N}$, and curvature κ of the curve $\vec{r}(t) = (t^2, 2t, \ln(t))$ when $t = 4$.

(10) For what value of x is the curvature of the curve $y = e^x$ maximized? What is the limit of the curvature as $x \to \infty$?

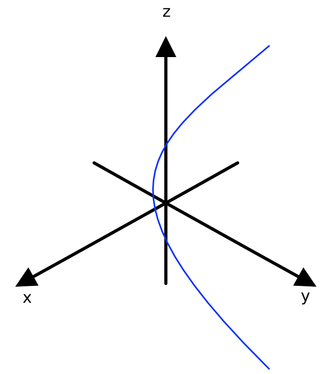
(11) Two graphs are shown below; one is a curve $y = f(x)$ and the other is the curvature $\kappa(x)$ of that curve. Identify which is which.



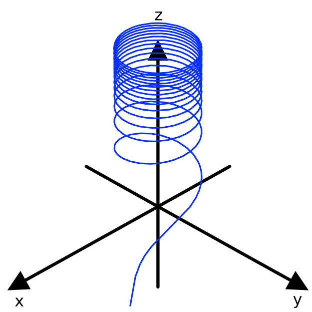
I



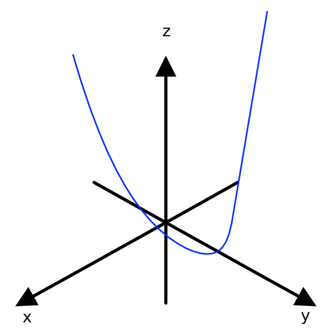
II



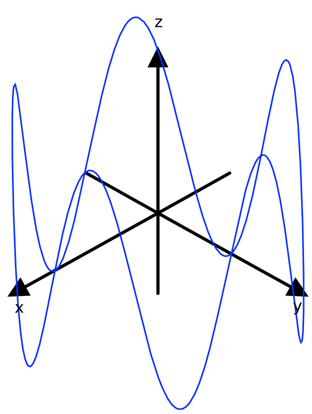
III



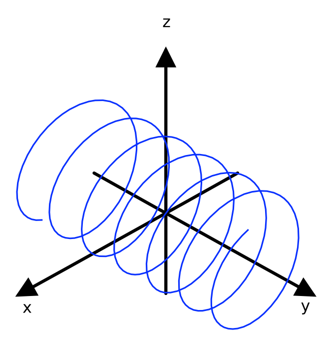
IV



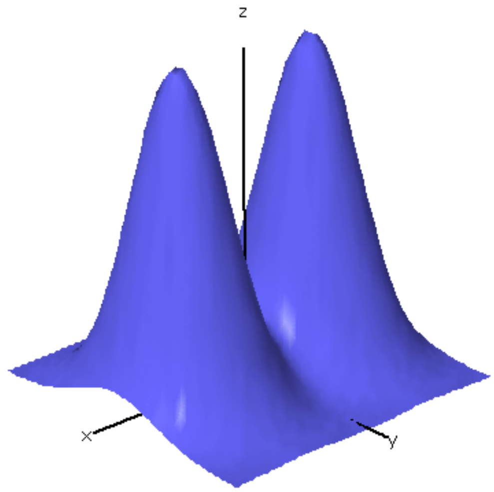
V



VI



(12) Sketch the contour map of the function whose graph is shown below.



(13) Sketch a contour map of the function $f = x^2 - y^2$.

(14) Sketch a contour map of the function $f = e^{y/x}$.

(15) Compute the following limits:

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2 + \sin^2(y)}{2x^2 + y^2}$ (b) $\lim_{(x,y)\to(0,0)} \dfrac{y^4 \sin(xy)}{x^2 + y^2}$

For Exercises 16 - 19, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(16) $f(x, y) = e^{xy} + xy$

(17) $f(x, y) = x^4$

(18) $f(x, y) = \dfrac{x+1}{y+1}$

(19) $f(x, y) = \ln(xy) + x^y$

For Exercises 20 and 21, calculate all four second-order partial derivatives of the function and verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

(20) $f(x, y) = x^2 + y^2$

(21) $f(x, y) = \cos(x + y)$

(22) The ideal gas law is $PV = cT$ where P is the pressure, V is the volume, T is the temperature and c is a constant for a given mass of some gas. Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

(Treat each variable as a function of the other two.)

(23) Find the linearization $L(x, y)$ of the function $f(x, y) = x\sqrt{y}$ at the point $(1, 4)$.