Homework 3, due June 6th

(1) Find the equation of the plane that is tangent to the surface $z = y\ln(x)$ at the point $(1, 4, 0)$.

(2) Use the linearization of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ to approximate $f(1.95, 1.05)$.

(3) Use the chain rule to compute $\dfrac{dz}{dt}$ if $z = x^2y + xy^2$, $x = 2 + t^4$, $y = 1 - t^3$.

(4) Use the chain rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ for $z = x^2 + xy + y^2$, $x = s + t$, $y = st$.

(5) Use the chain rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ for $z = x/y$, $x = se^t$, $y = 1 + se^{-t}$.

(6) Suppose that $f(x, y)$ is a differentiable function, and $g(r, s) = f(2r - s, s^2 - 4r)$. Compute $\dfrac{\partial g}{\partial r}|_{(r=1,s=2)}$ and $\dfrac{\partial g}{\partial s}|_{(r=1,s=2)}$ from the table of values below:

	f	g	$\partial f/\partial x$	$\partial f/\partial y$
$(0, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

(7) Compute the gradient of the function $f(x, y) = x^2e^y$.

(8) Compute the value of the gradient of the function $f(x, y, z) = \sin(xyz)$ at the point $(1, 1, \pi/4)$.

(9) Compute the directional derivative of $f(x, y) = x^2 + y^2 + 1$ at the point $(1, 1)$ in the direction $\vec{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

(10) Compute the directional derivative of $f(x, y) = x^2y^3 - y^4$ at the point $(2, 1)$ in the direction of the ray $\theta = \pi/4$.

(11) Compute the directional derivative of $f(x, y, z) = x^2e^{yz}$ at the point $(1, 1, 1)$ in the direction of $\vec{u} = (1, 1, 1)$.

(12) Find all of the directions in which the directional derivative of $f(x, y) = \sin(xy) + x^2$ at the point $(1, 0)$ has the value 1.

(13) Find the local maxima and minima of $f(x, y) = x^3y + 12x^2 - 8y$.

(14) Find the local maxima and minima of $f(x, y) = e^{4y - x^2 - y^2}$.

(15) Find three positive numbers x, y, z whose sum is 100 such that x^2y^2z is a maximum using the elimination method (i.e. not Lagrange multipliers).

(16) Find the maxima and minima of the function $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

(17) Find the points on the circle $x^2 + y^2 = 100$ which are closest to and farthest from the point $(2, 3)$.

(18) Suppose that a, b, and c are positive parameters. Using Lagrange multipliers, find formulae for the x,y,z in terms of a, b, and c which maximize $x^a y^b z^c$ subject to the constraint $x + y + z = 100$.

(19) Find the maximum value of $f(x_1, x_2, \ldots, x_n) = \sqrt[n]{x_1 x_2 \cdots x_n}$ subject to the constraint $x_1 + x_2 + \ldots + x_n = c$, where c is a constant. You can assume that the x_i are non-negative.

This implies the famous result that the geometric mean is always less than or equal to the arithmetic mean for $x_i \geq 0$, i.e.

$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \ldots + x_n}{n}.$$

(20) Some of the genes in our DNA are present in the human population in slightly varying forms, called alleles. Under some assumptions, the fraction of each form present will obey the Hardy-Weinberg Law. For the particular case of three alleles, present with frequencies a, b, and c, the Hardy-Weinberg Law says that the fraction of individuals with two different alleles (we have two copies of each gene) will be $2ab + 2ac + 2bc$. Given that these frequencies are constrained by the condition $a+b+c = 1$, show that the fraction of individuals with different alleles is at most $2/3$.