Assignment 4, due Friday, June 20th

(1) Estimate the volume of the solid bounded by \( z = 0 \), \( z = x + 2y^2 \), and within the rectangle \( x \in [0, 2] \), \( y \in [0, 4] \) by using a Riemann sum with \( n = m = 2 \) and use the value of the function in the lower-right corner of each sub-rectangle. Repeat the estimate using the midpoint of each rectangle - which of these do you expect to be more accurate?

(2) Estimate the volume of Angel Island, CA, in cubic feet by using the midpoint rule with a 3 by 3 subdivision as shown on the following map. The squares in the subdivision are 2000 feet square, and elevations are also shown in feet.

(3) Find the value of the integral \( \int \int_R 2 \, dA \), where \( R = \{(x, y) \mid -3 \leq x \leq 3, \ -2 \leq y \leq 2 \} \) by identifying it as the volume of a solid.

(4) Find the value of the integral \( \int \int_R 8 - x \, dA \), where \( R = \{(x, y) \mid 4 \leq x \leq 8, \ -1 \leq y \leq 1 \} \) by identifying it as the volume of a solid.

(5) Calculate the following double integrals

\[
(6) \quad \int_1^2 \int_0^1 \frac{ye^y}{x} \, dy \, dx \\
(7) \quad \int \int_R 2xye^{x^2y} \, dA, \ R = [0, 2] \times [0, 1].
\]
(8) Find the volume of the solid bounded by the planes \( x = 4, \ y = 2, \) the coordinate planes, and the elliptic paraboloid \( z = 2 + (x - 1)^2 + 8y^2. \)

(9) Compute the average value of the function \( f(x, y) = e^x \sqrt{y + e^x} \) on the rectangle \( R = [0, 1] \times [0, 2]. \)

(10) Compute the average value of the function \( f(x, y) = xy \) on the triangle with vertices \((0, 0), \ (3, 0), \) and \((1, 2). \)

(11) Compute the integral \( \iint_R (x + y) \, dA \) where \( R \) is the region bounded by \( y = \sqrt{x} \) and \( y = x^2. \)

Reverse the order of integration of the following two double integrals after sketching the region of integration. Note that you are not required to evaluate the resulting integral.

\[
(12) \quad \int_0^4 \int_{\sqrt{16-y^2}}^0 f(x, y) \, dx \, dy \quad \quad \quad (13) \quad \int_0^1 \int_{2x}^2 f(x, y) \, dy \, dx
\]

(14) Sketch the region of integration of the polar integral \( \int_0^{\pi/2} \int_0^{4 \cos(\theta)} r \, dr \, d\theta \), and then compute its value.

(15) Compute the volume of the solid bounded by the paraboloid \( z = 10 - 3x^2 - 3y^2 \) and the plane \( z = 4 \) by using polar coordinates.

(16) Compute the following sum of integrals by sketching the total region of integration and then switching to polar coordinates:

\[
\int_0^1 \int_{1/\sqrt{2}}^{\sqrt{1-x^2}} xy \, dy \, dx + \int_1^{\sqrt{2}} \int_0^x xy \, dy \, dx + \int_0^2 \int_{\sqrt{2}}^{\sqrt{1-x^2}} xy \, dy \, dx.
\]

(17) Find the center of mass of a lamina with density \( \rho = cxy \), if the lamina is the rectangle \([0, a] \times [0, b] \), where \( a, b, \) and \( c \) are all positive constants.

(18) Compute the value of the triple integral \( \int_0^2 \int_2^{2y} \int_0^x 2xyz \, dz \, dx \, dy. \)

(19) Compute the integral \( \iiint_\Omega x \, dV \) where \( \Omega \) is the region \( x \leq 4, \ x \geq 4y^2 + 4z^2. \)

(20) Find the center of mass of the solid bounded by the planes \( x = 0, \ z = 0, \ x + z = 1, \) and the parabolic cylinder \( z = 1 - y^2 \) with density \( \rho(x, y, z) = 4. \)

A picture of the boundaries of this solid is shown below.
(21) Find the volume of a wedge cut from a ball of radius $R$ by two planes which intersect on a diameter of the ball at an angle of $\pi/6$. (This volume is like a section of an orange.)

(22) Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation $x = (u^2 - v^2)/2$, $y = (u^2 + v^2)/2$.

(23) Use the transformation $x = \sqrt{2}u - \sqrt{2/3}v$, $y = \sqrt{2}u + \sqrt{2/3}v$ to compute the integral $\int \int_R (x^2 - xy + y^2)^{1/2} \, dA$ where $R$ is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. 