Assignment 4, due Friday, June 20th

(1) Estimate the volume of the solid bounded by $z = 0$, $z = x + 2y^2$, and within the rectange $x \in [0, 2]$, $y \in [0, 4]$ by using a Riemann sum with $n = m = 2$ and use the value of the function in the lower-right corner of each sub-rectangle. Repeat the estimate using the midpoint of each rectangle - which of these do you expect to be more accurate?

(2) Estimate the volume of Angel Island, CA, in cubic feet by using the midpoint rule with a 3 by 3 subdivision as shown on the following map. The squares in the subdivision are 2000 feet square, and elevations are also shown in feet.

(3) Find the value of the integral $\int\int_R 2\, dA$, where $R = \{(x, y)| -3 \leq x \leq 3,\ -2 \leq y \leq 2\}$ by identifying it as the volume of a solid.

(4) Find the value of the integral $\int\int_R 8 - x\ dA$, where $R = \{(x, y)|4 \leq x \leq 8,\ -1 \leq y \leq 1\}$ by identifying it as the volume of a solid.

(5) Calculate the following double integrals

(6) $\displaystyle\int_1^2 \int_0^1 \frac{ye^y}{x}\, dy\, dx$

(7) $\displaystyle\int\int_R 2xye^{x^2 y}\, dA$, $R = [0, 2] \times [0, 1]$.

(8) Find the volume of the solid bounded by the planes $x = 4$, $y = 2$, the coordinate planes, and the elliptic paraboloid $z = 2 + (x - 1)^2 + 8y^2$.

(9) Compute the average value of the function $f(x, y) = e^x\sqrt{y + e^x}$ on the rectangle $R = [0, 1] \times [0, 2]$.

(10) Compute the average value of the function $f(x, y) = xy$ on the triangle with vertices $(0, 0)$, $(3, 0)$, and $(1, 2)$.

(11) Compute the integral $\displaystyle\int\int_R (x + y)\ dA$ where R is the region bounded by $y = \sqrt{x}$ and $y = x^2$.

Reverse the order of integration of the following two double integrals after sketching the region of integration. Note that you are **not** required to evaluate the resulting integral.

(12) $\displaystyle\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} f(x, y)\, dx\, dy$

(13) $\displaystyle\int_0^1 \int_{2x}^{2} f(x, y)\, dy\, dx$

(14) Sketch the region of integration of the polar integral $\displaystyle\int_0^{\pi/2} \int_0^{4\cos(\theta)} r\ dr\ d\theta$, and then compute its value.

(15) Compute the volume of the solid bounded by the paraboloid $z = 10-3x^2-3y^2$ and the plane $z = 4$ by using polar coordinates.

(16) Compute the following sum of integrals by sketching the total region of integration and then switching to polar coordinates:

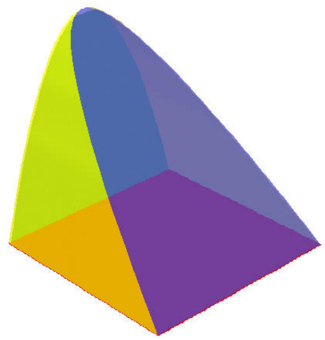
$$\int_{1/\sqrt{2}}^{1} \int_{\sqrt{1-x^2}}^{x} xy\ dy\ dx + \int_{1}^{\sqrt{2}} \int_{0}^{x} xy\ dy\ dx + \int_{\sqrt{2}}^{2} \int_{0}^{\sqrt{4-x^2}} xy\ dy\ dx.$$

(17) Find the center of mass of a lamina with density $\rho = cxy$, if the lamina is the rectangle $[0, a] \times [0, b]$, where a, b, and c are all positive constants.

(18) Compute the value of the triple integral $\displaystyle\int_0^2 \int_y^{2y} \int_0^x 2xyz\ dz\ dx\ dy$.

(19) Compute the integral $\displaystyle\int\int\int_\Omega x\ dV$ where Ω is the region $x \leq 4$, $x \geq 4y^2+4z^2$.

(20) Find the center of mass of the solid bounded by the planes $x = 0$, $z = 0$, $x + z = 1$, and the parabolic cylinder $z = 1 - y^2$ with density $\rho(x, y, z) = 4$. A picture of the boundaries of this solid is shown below.



(21) Find the volume of a wedge cut from a ball of radius R by two planes which intersect on a diameter of the ball at an angle of $\pi/6$. (This volume is like a section of an orange.)

(22) Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation $x = (u^2 - v^2)/2$, $y = (u^2 + v^2)/2$.

(23) Use the transformation $x = \sqrt{2}u - \sqrt{2/3}v$, $y = \sqrt{2}u + \sqrt{2/3}v$ to compute the integral $\int\int_R (x^2 - xy + y^2)^{1/2}\, dA$ where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$.