Homework 5, due Friday June 27

(1) Sketch the vector field $\vec{F} = (y, 1)$ in the square $x \in [-2, 2]$, $y \in [-2, 2]$.

(2) Sketch the vector field $\vec{F} = (x - y)\vec{i} + x\vec{j}$ in the square $x \in [-2, 2]$, $y \in [-2, 2]$.

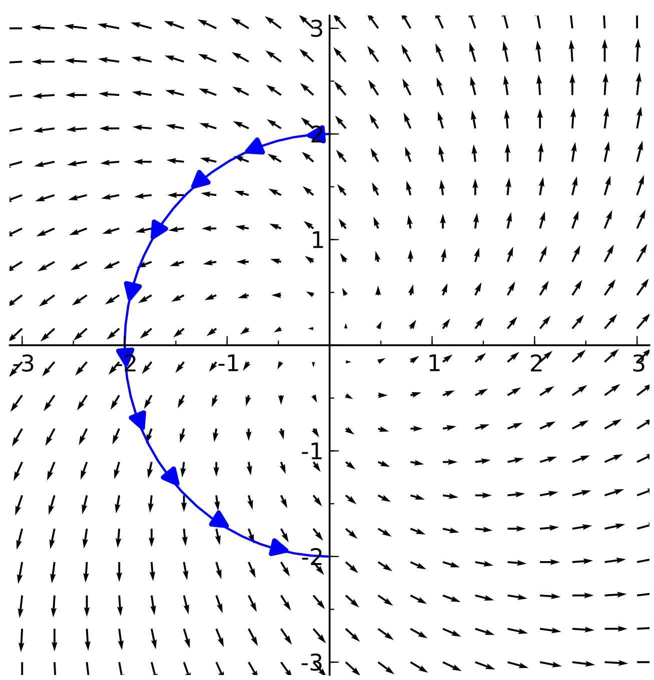
(3) Match the vector field plots on the third page to the following vector fields (as usual the x-axis is the horizontal axis in these plots):
  (I) $\vec{F} = (x - 1, x + 3)$
  (II) $\vec{F} = (y, x)$
  (III) $\vec{F} = (\sin(x), 1)$
  (IV) $\vec{F} = (y, 1/x)$

(4) Compute the scalar line integral $\int_C y \; ds$ where C is the curve $x = y^2$ for $y \in [0, 3]$.

(5) Compute the scalar line integral $\int_C x/y \; ds$ where C is the curve $x = t^3$, $y = t^4$, $t \in [1/2, 1]$.

(6) Compute the scalar line integral $\int_C xy^3 \; ds$ where C is the curve $x = 3\sin(t)$, $y = 3\cos(t)$, $z = 4t$ for $t \in [0, \pi/2]$.

(7) Is the vector line integral $\int_C \vec{F} \cdot \; d\vec{s}$ positive, negative, or zero for the $\vec{F}$ and C shown below? (The curve C is in blue.)



(8) Compute the vector line integral $\int_C \vec{F} \cdot \; d\vec{s}$ where $\vec{F} = (yz, xz, xy)$ and C is the curve $\vec{r}(t) = (t, t^2, t^3)$ for t in $[0, 1]$.

(9) Find a function f such that $\nabla f = \vec{F}$ where $\vec{F} = (4xy + \ln(x), 2x^2)$. Use this evaluate $\int_C \vec{F} \cdot \, d\vec{s}$ where C is a curve beginning at $(1, 0)$ and ending at $(2, 2)$ along which $x > 0$.

(10) Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (\sqrt{x} + y^3, x^2 + \sqrt{y})$ and C is the graph of $y = \sin(x)$ from $(0, 0)$ to $(\pi, 0)$ and the line segment from $(\pi, 0)$ to $(0, 0)$.

(11) Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (e^x + x^2y, e^y - xy^2)$ and C is the circle $x^2 + y^2 = 16$ (oriented positively).

(12) Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ directly and with Green's Theorem if $\vec{F} = (xy^2, x^3)$, and C is the rectangle (oriented positively) $(0, 0)$, $(2, 0)$, $(2, 3)$, and $(0, 3)$.

(13) Show that the area of a polygon with vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ (in counterclockwise order) is

$$A = \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \ldots + (x_{n-1}y_n - x_ny_{n-1}) + (x_ny_1 - x_1y_n)]$$

by using Green's theorem and calculating the line integral of $(-y/2, x/2)$ along each line segment..

(14) Calculate the curl and divergence of the vector field $\vec{F} = (2, x + yz, xy - \sqrt{z})$.

(15) Calculate the curl and divergence of the vector field $\vec{F} = (\cos(yz), 0, -\sin(xy))$.

(16) Construct a potential function for the vector field $(e^z, 6, xe^z)$.

(17) Is there a vector field $\vec{G}$ such that $\nabla \times \vec{G} = (xy^2, yz^2, zx^2)$? Explain why or why not.

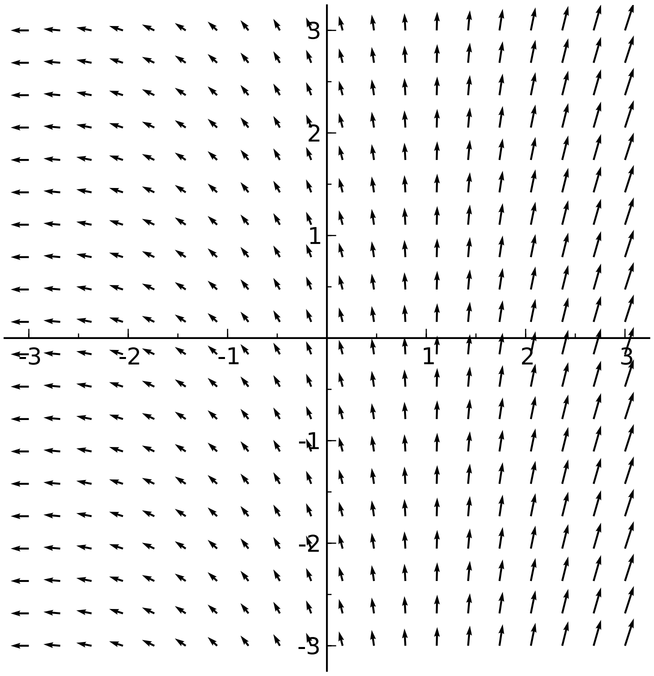
(18) Prove that $\nabla \cdot (f\vec{F}) = f\nabla \cdot \vec{F} + \vec{F} \cdot \nabla f$ if f is a scalar function of (x, y, z) with continuous partial derivatives and $\vec{F}$ is a vector field whose components also have continuous partial derivatives.

(19) If $\vec{r} = (x, y, z)$, $r = |\vec{r}|$, and $\vec{F} = \vec{r}/r^p$ (p is a real number), compute $\nabla \cdot \vec{F}$. Is there a value of p for which $\nabla \cdot \vec{F} = 0$?

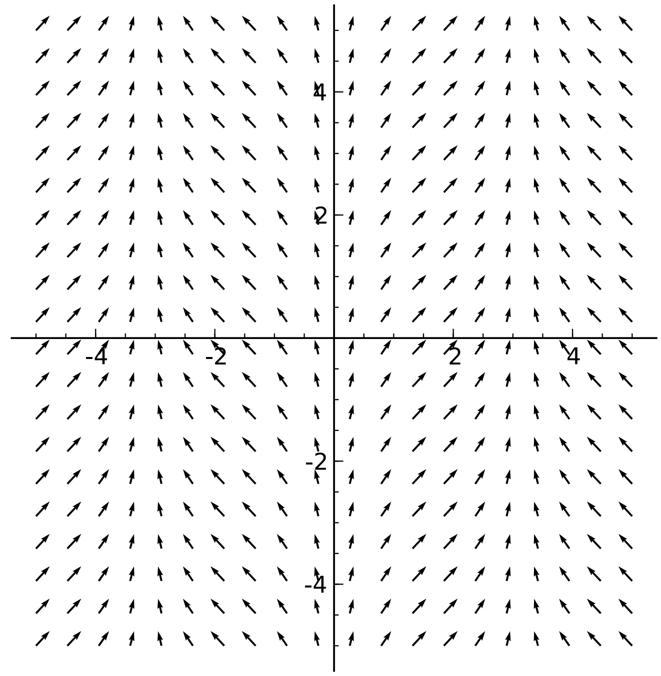
(20) Parameterize the plane which contains the point $(1, 1, 1)$ and which is parallel to the vectors $(1, 0, -1)$ and $(1, -1, 0)$.

(21) Parameterize the lower half of the ellipsoid $x^2/9 + y^2/4 + z^2 = 1$.

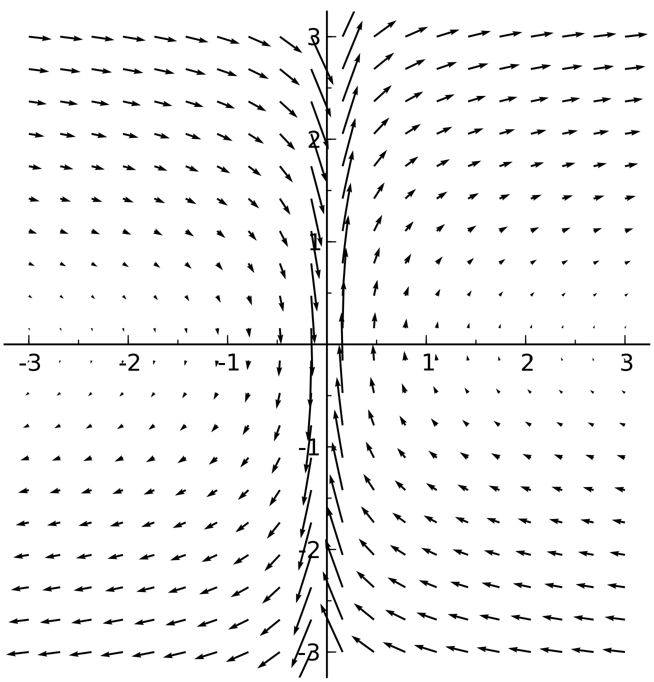
(22) Compute the surface area of the part of the plane $3x + 2y + z = 6$ that is inside the cylinder $x^2 + y^2 = 16$.



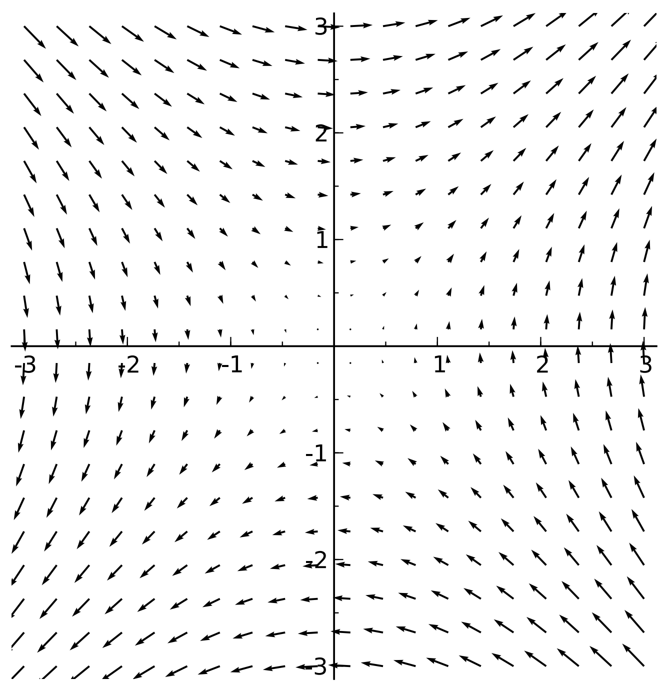
A.



B.



C.



D.