Math 4230 Practice final exam.

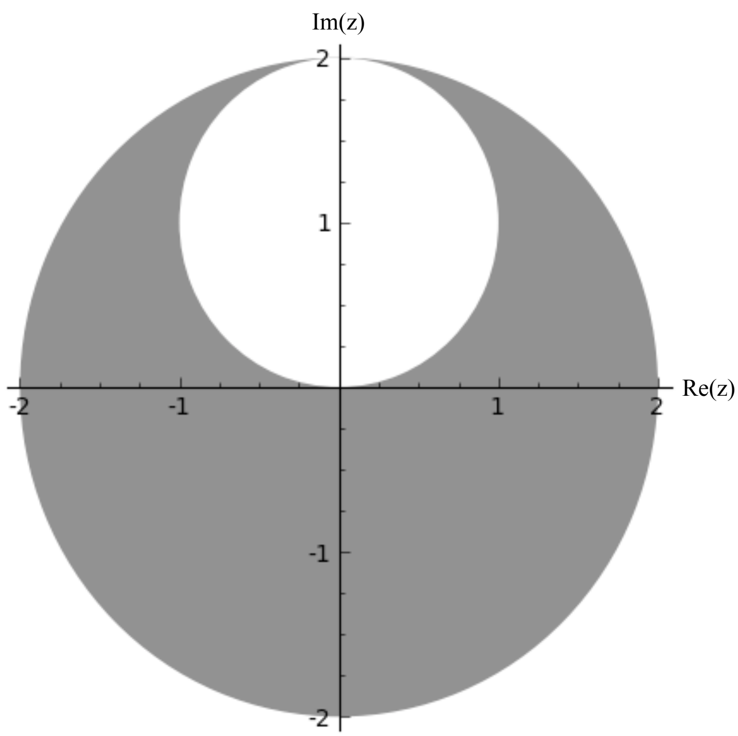
(1) Find all the complex solutions to the equation $\cos(z) = \sin(z)$.

(2) Without computing the integral exactly, show that

$$\left| \int_\Gamma \frac{1}{z^2 - i}\, dz \right| < \frac{\pi}{3}$$

if Γ is the counter-clockwise arc of the circle $|z| = 2$ starting at 2 and ending at $2i$.

(3) Find an analytic function that maps the shaded region below onto the upper half-plane. (One way is to compose a Mobius transformation with an exponential function.)



(4) Find the recursion relation for the coefficients of the power series solution $\sum_{n=0}^{\infty} a_n z^n$ around $z = 0$ which satisfies the differential equation

$$g'' - zg' - g = 0, \; g(0) = 0, \; g'(0) = 1.$$

(5) Construct a function $f(z)$ which has a pole of order 2 at the origin, a simple pole at ∞, and $Res(f; 0) = 1$.

(6) Compute the integral $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2+9)^2}\, dx$ by using the residue theorem.