Math 4326 Practice Midterm 1

This practice test should be roughly twice as long as the real one; some of the problems may be harder than on the exam. You need to justify all your answers.

The test will cover chapters 1 through 4.7, but not 2.6.

(1) Describe the set of $b$ for which the equation $Ax = b$ has a solution if

$$A = \begin{bmatrix} 3 & 17 & 0 \\ -6 & -34 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) Find bases for the nullspace and column space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

(3) Let $A$ be the matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Construct a $4 \times 2$ matrix $D$ using only 1s and 0s as entries such that $AD = I$. Is it possible that $CA = I$ for some $4 \times 2$ matrix $C$? Why or why not?

(4) Prove that the system

$$\begin{aligned} 3x_1 + 3x_2 + 4x_4 &= b_1 \\ x_1 + x_3 - x_4 &= b_2 \\ -x_2 + x_3 + x_4 &= b_3 \end{aligned}$$

always has a solution. You should not explicitly find the solution set.

(5) The numbers 20604, 53227, 25755, 20927, and 78421 are divisible by 17. Explain why the determinant $det \begin{bmatrix} 2 & 0 & 6 & 0 & 4 \\ 5 & 3 & 2 & 2 & 7 \\ 2 & 5 & 7 & 5 & 5 \\ 2 & 0 & 9 & 2 & 7 \\ 7 & 8 & 4 & 2 & 1 \end{bmatrix}$ is also divisible by 17.

(6) Suppose that $v_1$ and $v_2$ are linearly independent column vectors in $\mathbb{R}^4$. What are the possible values of the rank of the matrix $v_1v_1^T + v_2v_2^T$?

(7) Extra credit: Let $\mathbb{P}_2$ be the vector space of polynomials in the variable $x$ with coefficients in $\mathbb{R}$ and degree at most 2. Find the matrix representation of the linear operator $T : \mathbb{P}_2 \to \mathbb{P}_2$ where $T(p) = \frac{dp}{dx} + p$ for $p \in \mathbb{P}_2$, with respect to the basis $\{1, x, x^2\}$.