Homework Assignment 2 for Math 5327. Due Friday, February 8th.

(1) Read 2.1 - 2.5.

(2) Ungraded problems; you do not need to hand these in, but I will assume that you have done them and that you will ask questions if there is something you do not completely understand:

2.1.2, 2.2.1a, 2.3.7 (why is the solution manual misleading for this problem?), 2.4.6.

(3) Construct a system of three linear equations in four unknowns x_1, x_2, x_3, x_4 which has the general solution

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + s \begin{pmatrix} -1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

where s and t are scalar parameters.

(4) Find the reduced echelon form of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix}$$

if the entries are considered as elements of the field $\mathbb{Z}/3$ (equivalence classes of integers modulo 3; for example, $1+2=3\equiv 0$ and $2+2=4\equiv 1$).

(5) Optional project (can be shown to class as a presentation): Suppose a set of five vectors $\{v_1, v_2, v_3, v_4, v_5\}$ in in $\mathbb{R}^3$ has the property that any three of the vectors are linearly independent. If we write the three components of each vector as $v_i = (a_i, b_i, c_i)$, prove that the five vectors

$$w_i = (a_i^2, b_i^2, c_i^2, a_i b_i, a_i c_i, b_i c_i)$$

are linearly independent in $\mathbb{R}^6$.

(6) Let V be the set of pairs of real numbers (x, y) and let $\mathbb{R}$ be the field of real numbers. If we define operations of addition and scalar multiplication as below, is V a vector space? Explain why or why not.
$(x_1, y_1) + (x_2, y_2) = (x_1, x_2)$ and
$c(x, y) = (cx, y)$.

(7) We can consider the positive real numbers, $\mathbb{R}^+$, to be a vector space over $\mathbb{R}$ in which the "addition" is multiplication. What is the "zero" vector (additive identity) in this vector space? How can the scalar multiplication be defined?

(8) Optional presentation: provide a brief (approximately 15-minute) overview of how linear systems of equations arise in the study of electric circuits (this could be based on section 2.6 in the text).