Homework Assignment 4 for Math 5327. Due Monday, February 25th, by 3 pm.
If you do not hand this in during class, put it under my door (SCC 172) -
DO NOT put it in my mailbox.

Read sections 4.5 and 4.8, and chapter 6.

Ungraded problems: 4.2.5, 4.3.2, 4.4.5, 4.5.8, 4.7.17, 6.1.11, 6.1.13.

(1) Let $P_3$ be the vector space of polynomials in the variable $x$ with coefficients in $\mathbb{C}$ and degree less than 3. Find the matrix representation of the linear operator $T : P_3 \to P_3$ where $T(p) = \frac{dp}{dx} + p$ for $p \in P_3$, with respect to the basis $\{1, x, x^2\}$.

(2) The numbers 20604, 53227, 25755, 20927, and 78421 are divisible by 17. Explain why the determinant $\begin{vmatrix} 2 & 0 & 6 & 0 & 4 \\ 5 & 3 & 2 & 2 & 7 \\ 2 & 5 & 7 & 5 & 5 \\ 2 & 0 & 9 & 2 & 7 \\ 7 & 8 & 4 & 2 & 1 \end{vmatrix}$ is also divisible by 17.

(3) Compute the determinant of the $n \times n$ matrix

$$A_n = \begin{pmatrix} x & a & a & \cdots & a \\ a & x & a & \cdots & a \\ a & a & x & \cdots & a \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ a & a & a & \cdots & x \end{pmatrix}$$

(4) Recall that Hermitian (or self-adjoint) matrices are defined by the property $A^T = \bar{A}$ where $\bar{A}$ is the complex conjugate of $A$ (this is sometimes expressed as $A^* = A$ where $A^*$ is the conjugate transpose of $A$). The $2 \times 2$ complex Hermitian matrices are a four-dimensional vector space over $\mathbb{R}$. Find three linearly independent matrices $\{s_1, s_2, s_3\}$ in this space with the following properties:
(a) $\det(s_i) = -1$
(b) Exactly two entries in each $s_i$ are zero.

(5) Suppose that $\{v_1, ..., v_n\}$ form a basis for a vector space, with $n \geq 2$. Show that $\{v_1 + v_2, v_2 + v_3, ..., v_{n-1} + v_n, v_n + v_1\}$ form a basis if and only if $n$ is odd. What happens if we use $\{v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n - v_1\}$ instead?

(6) Suppose that a $n \times n$ matrix $A$ has the property that $A^2 = 0$. What is the maximum possible rank of $A$? Generalize this to the case that $A^m = 0$ for $m > 2$.