Homework Assignment 5 for Math 5327. Due Wednesday, March 6, by 3 pm.
If you do not hand this in during class, put it under my door (SCC 172) -
DO NOT put it in my mailbox.

Read sections 5.1 - 5.7, 5.9 - 5.11, and 5.13.

Ungraded problems: 5.2.1, 5.2.2, 5.2.8, 5.3.4, 5.4.9.

(1)

(a) In the space P_2 of polynomials of degree less than 3, use the inner product

$$< p(x) \mid q(x) >= \int_0^1 p(x)q(x)dx$$

to find the angles of the triangle formed by the elements 1,x, x^2. Note that these elements are the points, not the edges, of the triangle. Verify (numerically) that your answers add up to π.

(b) The basis $\beta = \{1, x, x^2\}$ induces an isomorphism between P_2 and $\mathbb{R}^3$. Find a matrix A such that the corresponding inner product in $\mathbb{R}^3$ is $< p \mid q >= p^T A\, q$.

(2) Find orthogonal projection matrices P_W and $P_{\perp W}$ where W is the subspace of $\mathbb{R}^3$ spanned by the vectors $(1, 1, 0)$ and $(0, 1, 1)$. (I.e. the range of P_W is W, and the range of $P_{\perp W}$ is the subspace orthogonal to W.)

(3) Find an orthonormal basis for the subspace of $\mathbb{R}^4$ that is perpendicular to the vector $(1, 1, 1, 1)$.

(4) If a, b, c, and d are positive numbers and

$$v_1 = \begin{pmatrix} a \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} b \\ b \\ 0 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} c \\ c \\ c \\ c \end{pmatrix}, \quad v_4 = \begin{pmatrix} -d \\ -d \\ d \\ d \end{pmatrix}$$

what is the orthonormal basis that would be produced by the Gram-Schmidt algorithm?

(5) Construct a 2×2 matrix that projects onto the line $x_2 = ax_1$ along the x_2 direction, where a is a real parameter.

(6) Prove that if P is a nonzero projection matrix, then $|P|_2 \geq 1$, with equality if and only if P is an orthogonal projection. (Here $|P|_2$ means the induced matrix 2-norm.)