Homework Assignment 6 for Math 5327. Due Wednesday, April 3rd, by 3pm.

Read sections 7.1, 7.2, and 7.3.

Ungraded problems: 7.1.7, 7.2.6, 7.2.16, 7.3.10.

(1) A 7×7 matrix has the properties that $Range(A^4) \subset N(A)$ and $A^4 \neq 0$. What are the possible values for the rank of A? Provide examples of matrices with each of the possible ranks.

(2) P is an orthogonal projection matrix with a three-dimensional range and a two-dimensional nullspace. Describe as completely as possible the properties of the full SVD of P.

(3) Find the eigenvectors and eigenvalues of the matrix

$$\begin{bmatrix} -1 & 1 & 1 \\ -2 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

(4) Suppose T is a linear operator on an n-dimensional vector space V, and any matrix representation of T is diagonal with distinct diagonal values (i.e. no two diagonal elements are equal). Show that there are *exactly* 2^n invariant subspaces of V under the action of T.

(5) In several contexts, such as the theory of Markov chains, we deal with matrices with non-negative entries which have a constant row and column sum (i.e. for an n by n matrix with entries $a_{ij} \geq 0$ we have $\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = c$ for some number c).

(a) Show that for such a matrix, the eigenvalue of largest value is c.

(b) Find a counterexample to this if nonpositive entries are allowed.

(6) Describe a topic you would like to do your presentation on (if you have not already given one). This can be from the list on the course page, or something from the text that was not covered in class, or an application that involves significant use of linear algebra.