Homework Assignment 7 for Math 5327. Due Wednesday, April 17th, by 3pm.

Read sections 7.5 through 7.8. (We will skip Chapter 7.4, on systems of differential equations.)

Ungraded problems: 7.3.6, 7.3.12, 7.5.2, 7.7.2.

(1) Find an upper Hessenberg form for the matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 2 & 4 & 5 \end{bmatrix}$$

(2) Give examples of 3×3 real matrices with minimal polynomials $\lambda - 2$, $(\lambda - 2)^2$, and $(\lambda - 2)^3$.

(3) Investigate the eigenvalues of $n \times n$ matrices with random entries. Use entries from a normal distribution with mean $= 0$ and standard deviation $n^{-1/2}$ for $n = 8, 32, 128$ (at least; you are encouraged to experiment); plot the eigenvalues in the complex plane. Use at least 100 matrices of each type before trying to draw any conclusions. You do not have to prove anything, but you should formulate some hypotheses as clearly as you can.

(4) Classify up to similarity all of the 3×3 real matrix solutions to the equation $A^3 = I$.

(5) Suppose A is a $n \times m$ matrix and B is a $m \times n$ matrix, with $m > n$. What is the relationship between the characteristic polynomials of AB and BA?

(6) Are the matrices $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ unitarily equivalent? Prove your answer.

(7) Suppose that J_n are the $n \times n$ matrices with all entries equal to one. For example, $J_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Find the minimal and characteristic polynomials for the J_n, and prove that J_n is diagonalizable for all positive integer n.