Math 5327 Miscellaneous project/presentation suggestions

- (1) Explain what the "numerical range" of a matrix is. As an example compute the numerical range of $\begin{pmatrix} 0 & 0 & i \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.
- (2) Define and explain the resultant of two polynomials, and its properties. Go through at least one illustrative example.
- (3) Describe the Fourier-Motzkin algorithm, and show at least one example of its use.
- (4) Define and explain what the pseudospectrum of a matrix is, and why it can be a useful concept.
- (5) Explain what a Pfaffian is, and illustrate it with an example (the 4×4 case might be best). Prove Cayley's theorem on Pfaffians: the determinant of an antisymmetric matrix A of even order is the square of a homogeneous polynomial of degree n/2 in the entries of A. (I.e. $det(A) = P^2$.)
- (6) Define symplectic matrices and give some background on their importance. Prove that symplectic matrices have determinant 1.
- (7) Explain and prove Rellich's theorem: If $A(t) = \sum_{i=0}^{\infty} A_i t_i$ is an analytic matrix function of t, and the A_i are self-adjoint matrices, then the eigenvalues of A(t) are regular analytic functions of t.
- (8) The fast Fourier transform: what it is, how it works, what its used for.
- (9) What are small world matrices and what has been proven about them.
- (10) What does card shuffling have to do with matrix norms?
- (11) Describe how populations are modelled with stage-based matrices.
- (12) Use the SVD to investigate patterns in voting, for example from the United States Supreme Court.
- (13) Explain the Dodgson (aka Lewis Carroll) condensation method of computing determinants.