Homework Assignment Assignment 4, due Friday, February 15.

1) Read 4.1 - 4.5.

2) Do recommended problems: 3.7.8, 4.1.2, 4.2.1, 4.2.3, 4.2.12, 4.3.4.

3) Let $V$ be the set of pairs of real numbers $(x, y)$ and let $\mathbb{R}$ be the field of real numbers. If we define operations of addition and scalar multiplication as below, is $V$ a vector space? Explain why or why not.

$$
(x_1, y_1) + (x_2, y_2) = (x_1, x_2) \quad \text{and} \\
\alpha (x, y) = (\alpha x, y).
$$

4) Is the set of continuous, real-valued functions on $\mathbb{R}$ such that $f(-x) = -f(x)$ a vector space? Explain why or why not.

5) Find a basis $(v_1, v_2, v_3)$ for $\mathbb{R}^3$ such that none of the components of the $v_i$ are zero, and the determinant $Det(v_1|v_2|v_3) = 1$.

6) In a coordinate space $F^n$ ($F$ is a field) consider a set of $m \geq n+2$ nonzero vectors $v_i$. Prove that there are field elements $\alpha_1, \ldots, \alpha_m \in F$ such that $\sum_{i=1}^{m} \alpha_i v_i = 0$ and $\sum_{i=1}^{m} \alpha_i = 0$ (with not all $\alpha_i = 0$).

7) Suppose that $X$, $Y$, and $Z$ are subspaces of a vector space $V$. Is the identity

$$
dim(X+Y+Z) = dim(X) + dim(Y) + dim(Z) - dim(X \cap Y) - dim(X \cap Z) - dim(Y \cap Z) + dim(X \cap Y \cap Z)
$$

always true? If not, provide a counterexample. If so, prove it.