

Homework Assignment Assignment 4, due Friday, February 15.

- (1) Read 4.1 - 4.5.
- (2) Do recommended problems: 3.7.8, 4.1.2, 4.2.1, 4.2.3, 4.2.12, 4.3.4.
- (3) Let V be the set of pairs of real numbers (x, y) and let \mathbb{R} be the field of real numbers. If we define operations of addition and scalar multiplication as below, is V a vector space? Explain why or why not.
 $(x_1, y_1) + (x_2, y_2) = (x_1, x_2)$ and
 $c(x, y) = (cx, y)$.
- (4) Is the set of continuous, real-valued functions on \mathbb{R} such that $f(-x) = -f(x)$ a vector space? Explain why or why not.
- (5) Find a basis (v_1, v_2, v_3) for \mathbb{R}^3 such that none of the components of the v_i are zero, and the determinant $\text{Det}(v_1|v_2|v_3) = 1$.
- (6) In a coordinate space F^n (F is a field) consider a set of $m \geq n+2$ nonzero vectors v_i . Prove that there are field elements $\alpha_1, \dots, \alpha_m \in F$ such that $\sum_{i=1}^m \alpha_i v_i = 0$ and $\sum_{i=1}^m \alpha_i = 0$ (with not all $\alpha_i = 0$).
- (7) Suppose that X, Y , and Z are subspaces of a vector space V . Is the identity $\dim(X+Y+Z) = \dim(X) + \dim(Y) + \dim(Z) - \dim(X \cap Y) - \dim(X \cap Z) - \dim(Y \cap Z) + \dim(X \cap Y \cap Z)$ always true? If not, provide a counterexample. If so, prove it.