

Math 5327 Assignment 5, due Friday, February 22nd.

- (1) Reading: 4.7 - 4.8. We may not cover all of 4.8 this week, but read as much as you can.
- (2) Recommended problems: 4.4.4, 4.4.5, 4.4.7, 4.4.11, 4.5.17, 4.5.18, 4.7.2. These are mostly from 4.4 and 4.5 because of the extreme importance of understanding bases and rank.
- (3) Let x and y be n -dimensional column vectors (i.e. they are $n \times 1$ matrices). If we define a $n \times n$ matrix A by $a_{ij} = x_i + y_j$, show that $\text{rank}(A) \leq 2$.
- (4) Suppose that $\{v_1, \dots, v_n\}$ form a basis for a vector space, with $n \geq 2$. Show that $\{v_1 + v_2, v_2 + v_3, \dots, v_{n-1} + v_n, v_n + v_1\}$ form a basis if and only if n is odd. What happens if we use $\{v_1 - v_2, v_2 - v_3, \dots, v_{n-1} - v_n, v_n - v_1\}$ instead?
- (5) Suppose that a $n \times n$ matrix A has the property that $A^2 = 0$. What is the maximum possible rank of A ? Generalize this to the case that $A^m = 0$ for $m > 2$.