Math 5327 Spring 2008 Assignment 6, due February 29th.

(1) Reading: 7.1.

(2) Recommended problems: 4.7.13, 4.8.6, 4.8.11, 4.8.12, 4.9.3, 4.9.8, 7.1.15.

(3) In several contexts, such as the theory of Markov processes or Sodoku puzzles, we deal with matrices with non-negative entries which have a constant row and column sum (i.e. for an n by n matrix with entries $a_{ij} \geq 0$ we have $\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = c$ for some number c). Show that for such a matrix, the eigenvalue of largest value is c. Find a counterexample to this if nonpositive entries are allowed.

(4) Suppose that $T : V \to V$ is a linear operator, $dim(V) = n$, and T has n eigenvectors with distinct eigenvalues λ_i ($\lambda_i \neq \lambda_j$ if $i \neq j$). Prove that T has **exactly** 2^n invariant subspaces.

(5) What can you determine about the spectrum of A if μ^2 is an eigenvalue of A^2?