Math 5327 Assignment 8, due Friday, March 28th.

(1) Read 5.6 - 5.9, and 5.13.

(2) Ungraded problems: 5.4.6, 5.4.12, 5.5.6, 5.5.11, 5.6.6, 5.6.9, 5.6.10, 5.6.14, 5.9.13, 5.9.17

(3) Find a set of three polynomials \( \{p_0(t), p_1(t), p_2(t)\} \) with real coefficients such that the degree of \( p_i = i \) and the set is orthonormal with respect to the inner product \( < f | g > = \int_0^2 f(t)g(t) \, dt \).

(4) Find the least squares ‘solution’ to

\[
\begin{bmatrix}
2 \\
3 \\
4
\end{bmatrix}
x = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]

(5) Let \( x \) be a \( m \)-vector and \( A \) a \( m \times n \) matrix over \( \mathbb{C} \). For each of the following, prove the inequality and give an example of a matrix or vector for which equality is achieved:

(a) \( |x|_\infty \leq |x|_2 \)
(b) \( |A|_\infty \leq \sqrt{n}|A|_2 \)
(c) \( |A|_2 \leq \sqrt{m}|A|_\infty \)