Math 5327 Assignment 9, due Friday, April 4th.

(1) Read 5.10 - 5.14.

(2) Ungraded problems: 5.9.17, 5.10.8, 5.10.11, 5.11.5, 5.11.13, 5.12.8, 5.13.15.

(3) Two matrices $A, B \in \mathbb{C}^{m\times m}$ are *unitarily equivalent* if there is a unitary matrix $Q \in \mathbb{C}^{m\times m}$ such that $A = QBQ^*$. If A and B are unitarily equivalent, are the singular values of A and B equal? What about the converse?

(4) Construct a projection matrix P_1 which has a 1-dimensional nullspace and a 2-dimensional range, but which is not an orthogonal projection. Then find orthogonal projection matrices P_2 and P_3 such that the range of P_2 equals the range of P_1, and the nullspace of P_3 equals the nullspace of P_1.

(5) Consider the set $\mathcal{W}$ of 2×2 unitary matrices with determinant equal to 1 and whose diagonal entries are real and equal, i.e. if $A \in \mathcal{W}$ then $det(A) = 1$, $A^*A = I = AA^*$ and $A = \begin{pmatrix} a & b + c\ i \\ d + f\ i & a \end{pmatrix}$ where $a, b, c, d, f \in \mathbb{R}$ and $i^2 = -1$. What conditions must the entries satisfy?

(6) Presentations: Are you planning on working with others on your presentation? If so, who? What topics are you considering?

(7) Extra credit: Suppose A and B are $n \times n$ orthogonal projection matrices. Relate the eigenvalues of the sum $A + B$ to the singular values s_i of the orthogonal projection P of the range of A into the range of B (i.e., the singular values of $P|_{R(A)}$).