

Practice final for Math 5327.

(1) Find the Jordan normal form of the matrix $\begin{pmatrix} -2 & 0 & 1 \\ -1 & -2 & 2 \\ -1 & -1 & 1 \end{pmatrix}$. You do not have to explicitly compute the similarity matrix.

(2) Show that a $n \times n$ matrix A is normal if and only if $\text{tr}(A^*A) = \sum_{i=1}^n |\lambda_i|^2$ where λ_i is the i^{th} eigenvalue of A .

(3) Show that the eigenvalues of the tridiagonal Toeplitz matrices

$$B_n = \begin{pmatrix} a & b & 0 & 0 & \dots & 0 & 0 \\ c & a & b & 0 & \dots & 0 & 0 \\ 0 & c & a & b & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & a & b \\ 0 & 0 & 0 & 0 & \dots & c & a \end{pmatrix}$$

only depend on a and the product bc (instead of b and c individually).

(4) Is the set of trace-free (i.e. their trace is zero) linear transformations of \mathbb{C}^n a vector space? Justify your answer.

(5) Find the orthogonal projection matrix that projects onto the subspace of \mathbb{R}^4 satisfying the conditions $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 - x_2 = 0$.

(6) What is the set of λ for which $\lim_{n \rightarrow \infty} J_\lambda^n = 0$ if $J_\lambda = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$? Extra credit question: for which λ in this set does the 2-norm of J_λ^n decrease monotonically?

(7) Find the least squares solution to $\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 3 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(8) Suppose that a 8×8 matrix A has the property that $\text{Range}(A^4) \subset \text{Nullspace}(A^3)$, and $\dim(\text{Range}(A^3)) = 3$. What are the possible values of the rank of A ? What are the possible Jordan forms for A ?

(9) Extra credit: if P_M and P_N are orthogonal projectors onto subspaces M and N , respectively, show that the orthogonal projection onto $M \cap N$ can be written as $2P_M(P_M + P_N)^\dagger P_N$ where † denotes the pseudo-inverse.