(1) Find the Jordan normal form of the matrix \( \begin{pmatrix} -2 & 0 & 1 \\ -1 & -2 & 2 \\ -1 & -1 & 1 \end{pmatrix} \). You do not have to explicitly compute the similarity matrix.

(2) Show that a \( n \times n \) matrix \( A \) is normal if and only if \( \text{tr}(A^*A) = \sum_{i=1}^{n} |\lambda_i|^2 \) where \( \lambda_i \) is the \( i \)th eigenvalue of \( A \).

(3) Show that the eigenvalues of the tridiagonal Toeplitz matrices

\[
B_n = \begin{pmatrix}
a & b & 0 & 0 & \ldots & 0 & 0 \\
c & a & b & 0 & \ldots & 0 & 0 \\
0 & c & a & b & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & a & b \\
0 & 0 & 0 & 0 & \ldots & c & a
\end{pmatrix}
\]

only depend on \( a \) and the product \( bc \) (instead of \( b \) and \( c \) individually).

(4) Is the set of trace-free (i.e. their trace is zero) linear transformations of \( \mathbb{C}^n \) a vector space? Justify your answer.

(5) Find the orthogonal projection matrix that projects onto the subspace of \( \mathbb{R}^4 \) satisfying the conditions \( x_1 + x_2 + x_3 + x_4 = 0 \) and \( x_1 - x_2 = 0 \).

(6) What is the set of \( \lambda \) for which \( \lim_{n \to \infty} J^n_\lambda = 0 \) if \( J_\lambda = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \)? Extra credit question: for which \( \lambda \) in this set does the 2-norm of \( J^n_\lambda \) decrease monotonically?

(7) Find the least squares solution to \( \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 3 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \).

(8) Suppose that a \( 8 \times 8 \) matrix \( A \) has the property that \( \text{Range}(A^4) \subset \text{Nullspace}(A^3) \), and \( \dim(\text{Range}(A^3)) = 3 \). What are the possible values of the rank of \( A \)? What are the possible Jordan forms for \( A \)?

(9) Extra credit: if \( P_M \) and \( P_N \) are orthogonal projectors onto subspaces \( M \) and \( N \), respectively, show that the orthogonal projection onto \( M \cap N \) can be written as \( 2P_M(P_M + P_N)^\dagger P_N \) where \( \dagger \) denotes the pseudo-inverse.