

Practice midterm 2 for Math 5327. A sheet of notes and calculator will be allowed on the exam. This is longer than the actual exam will be.

- (1) Find the QR decomposition of $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 2 & 3 \end{pmatrix}$.
- (2) Suppose A is a 3×3 matrix whose SVD $U\Sigma V^*$ has the following properties: $U = V$, and $\sigma_1 = 2$, $\sigma_2 = 1$, $\sigma_3 = \frac{1}{2}$. What is the best rank-2 approximation to A ? Is it an orthogonal projection?
- (3) Show that we can define an inner product on $\mathbb{R}^{n \times n}$ by $(A, B) = \text{tr}(A^*B)$.
- (4) Find the orthogonal projection matrix that projects onto the subspace $x_1 + 2x_2 + 3x_3 = 0$ of \mathbb{R}^3 (with the standard inner product).
- (5) Construct the first Householder reflection matrix H_1 for $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 4 \end{pmatrix}$ -
i.e. H_1 should be a unitary matrix and the first column of H_1A should be $(\sqrt{14}, 0, 0)^T$.
- (6) Find the least squares solution to $\begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 4 \end{pmatrix} x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.