# Correlation & Regression
# Chapter 5

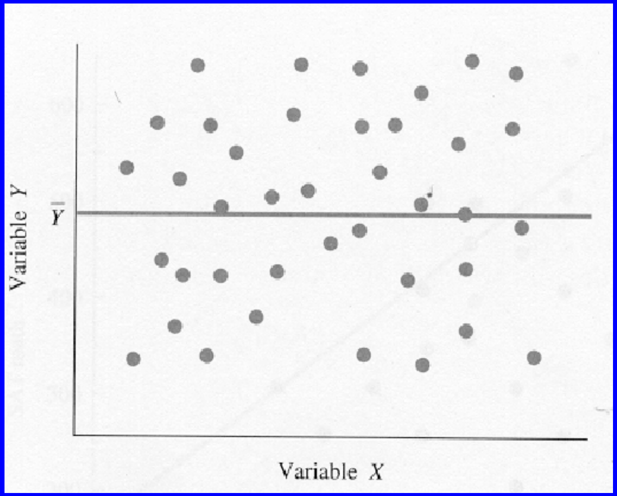
Correlation: Do you have a relationship?
Between two Quantitative Variables (measured on Same Person)

(1) If you have a relationship ($p<0.05$)?
(2) What is the Direction (+ vs. -)?
(3) What is the Strength ($r$: from –1 to +1)?

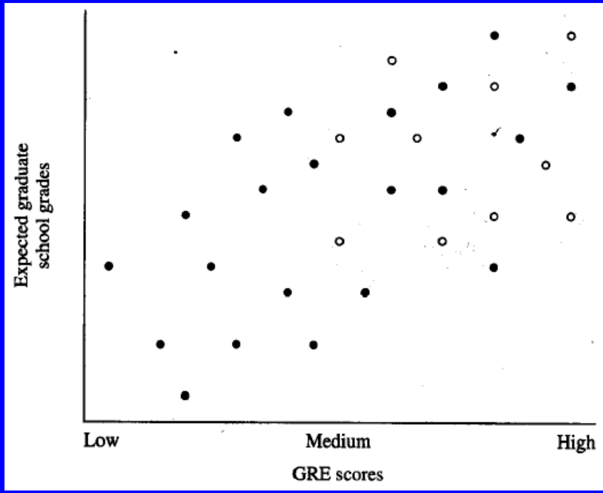
Regression: If you have a Significant Correlation:
How well can you Predict a subject's y-score if you know their X-score (and vice versa)
Are predictions for members of the Population as good As predictions for Sample members?

# Correlations measure LINEAR Relationships

No Relationship: r=0.0
Y-scores do not have a
Tendency to go up or down as
X-scores go up
You cannot Predict a person's
Y-value if you know his X-
Value any better than if you
Didn't know his X-score

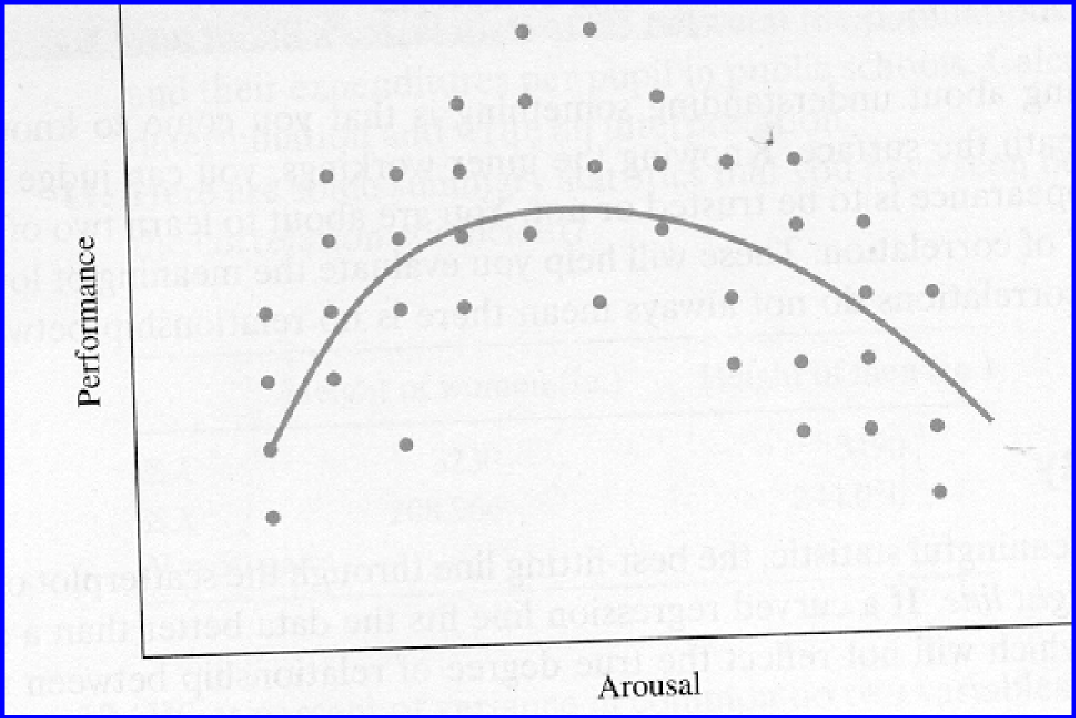



Positive Linear Relationship:
Y-scores tend to go up as
X-scores go up



# Correlations measure LINEAR Relationships, cont.

- There IS a relationship, but its not Linear
  - R=0.0, but that DOESN'T mean that the two variables are Unrelated



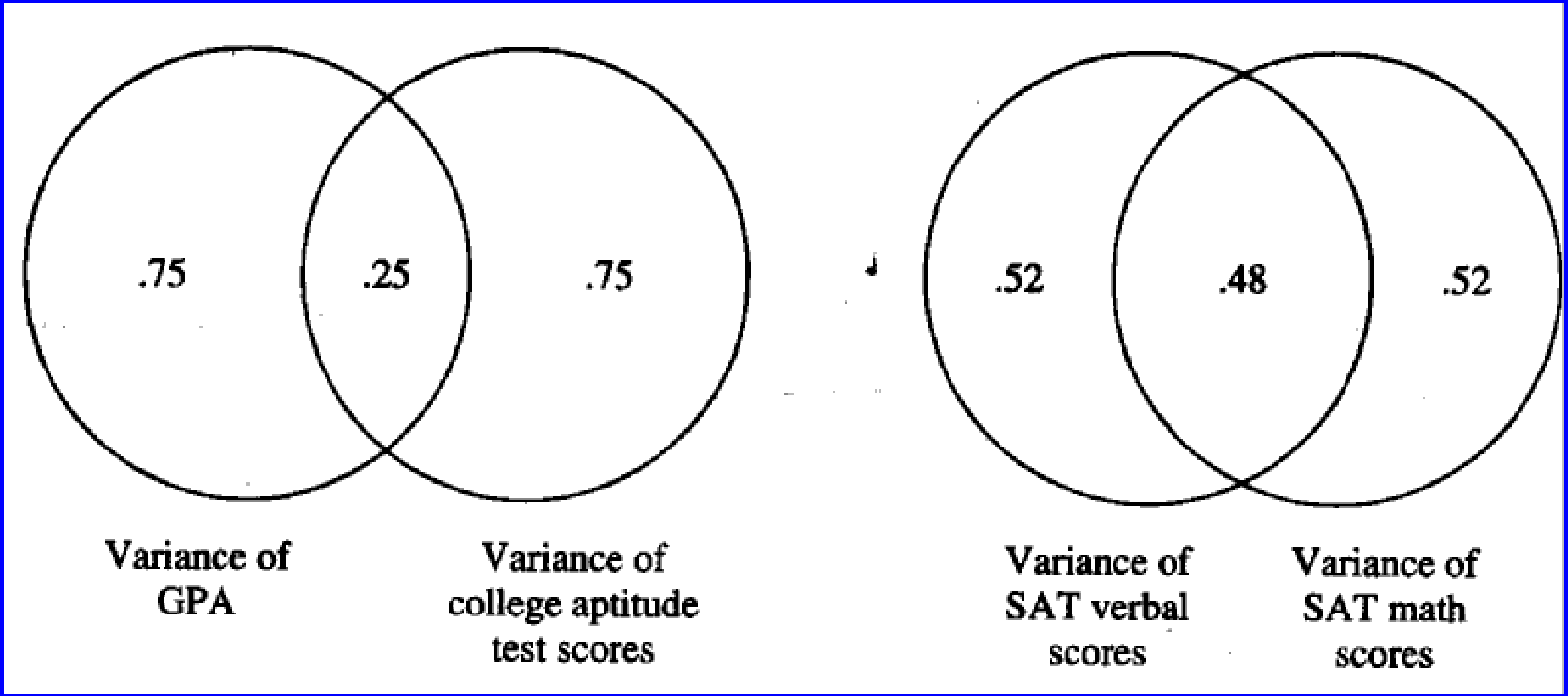
# Interpreting r-values

Coefficient of Determination – $r^2$:

Square of r-value

$r^2 * 100$ = Percent of Shared Variance; the Rest of the variance Is Independent of the other variable

r=0.50 r=0.6928



# Interpreting r-values

If the Coefficient of Determination between height and weight Is $r^2$=0.3 (r=0.9):

- 30% of variability in peoples weight can be Related to their height

- 70% of the difference between people in their of weight Is Independent of their height

- Remember: This does not mean that weight is partially Caused by height

  Arm and leg length have a high coefficient of Determination but a growing leg does not cause Your arm to grow

# IV & DV both Quantitative

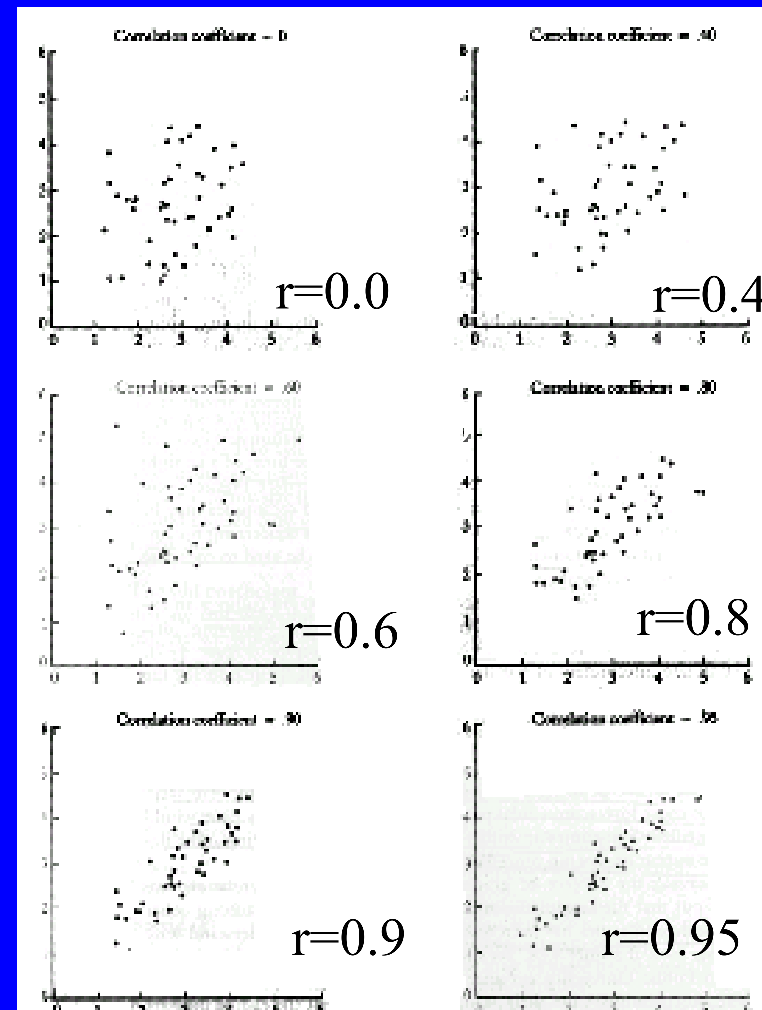
Correlation:

Each data point represents Two Measures from Same person.

1. Is There a Relationship?
2. What Direction is the Relationship?
3. How Strong is the Relationship?

-1 0 1

The stronger the relationship, the better you can predict one score if you know the other.

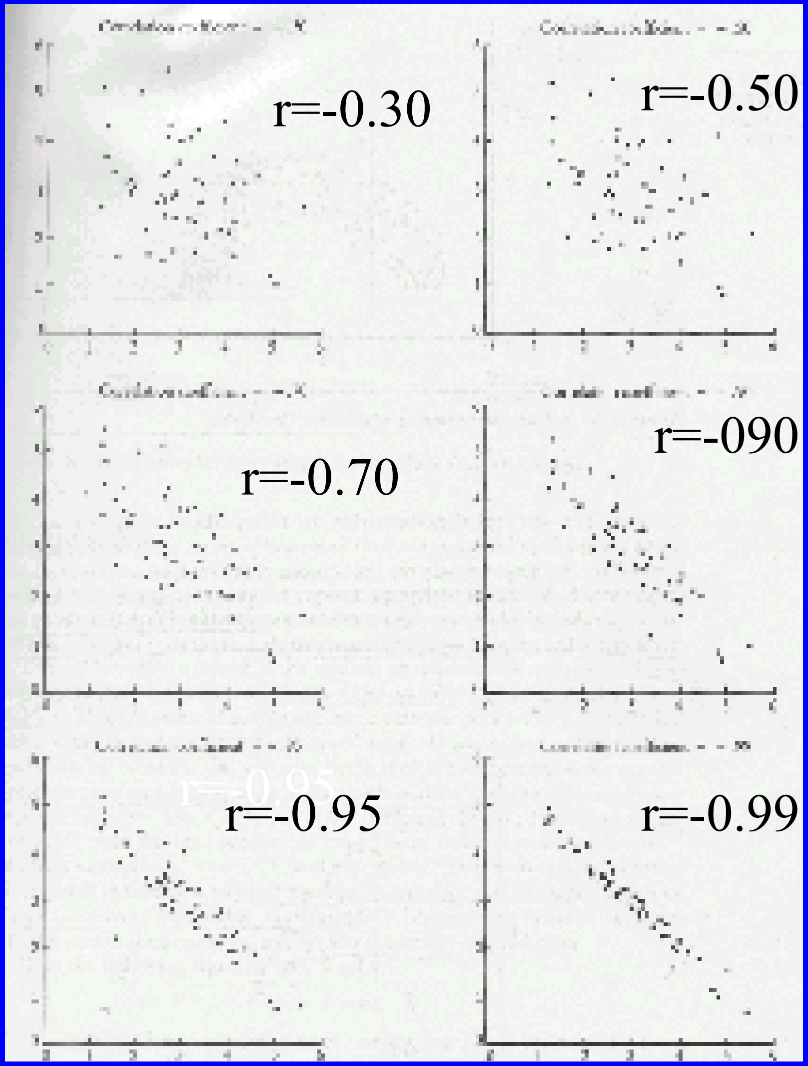


# Negative Correlation

Quasi-Independent Variable:
# of cigarettes/day

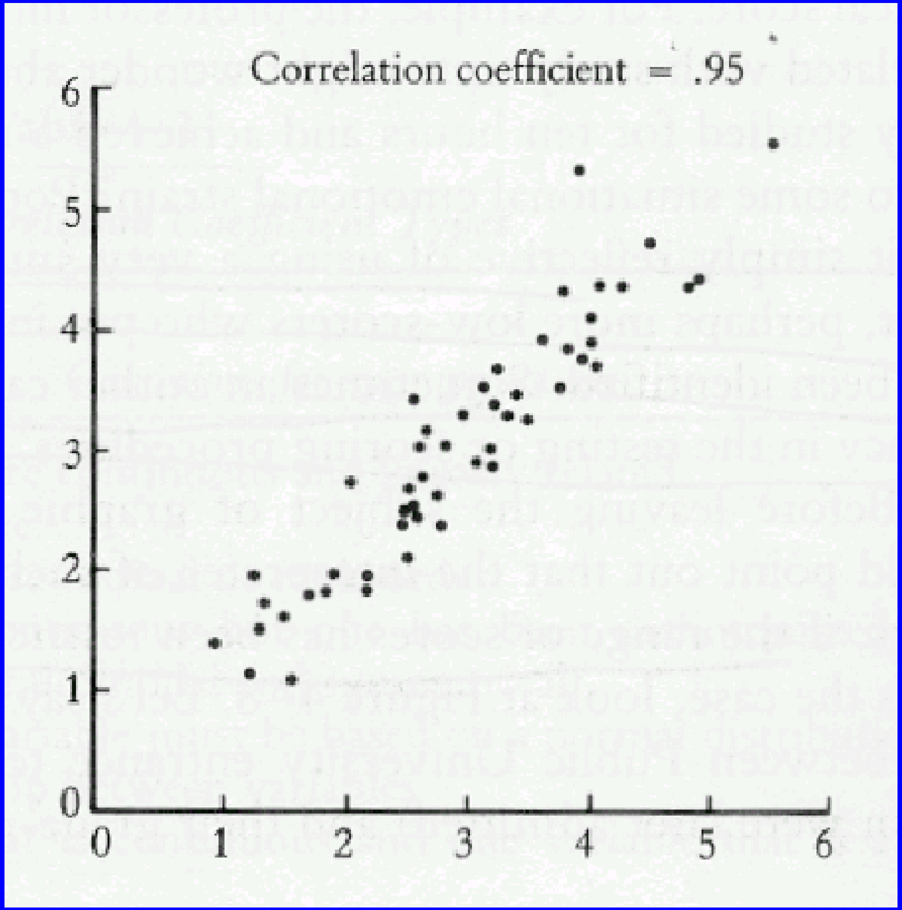
Dependent Variable:
Physical Endurance

The fatter the field, the weaker the correlation



# Correlations

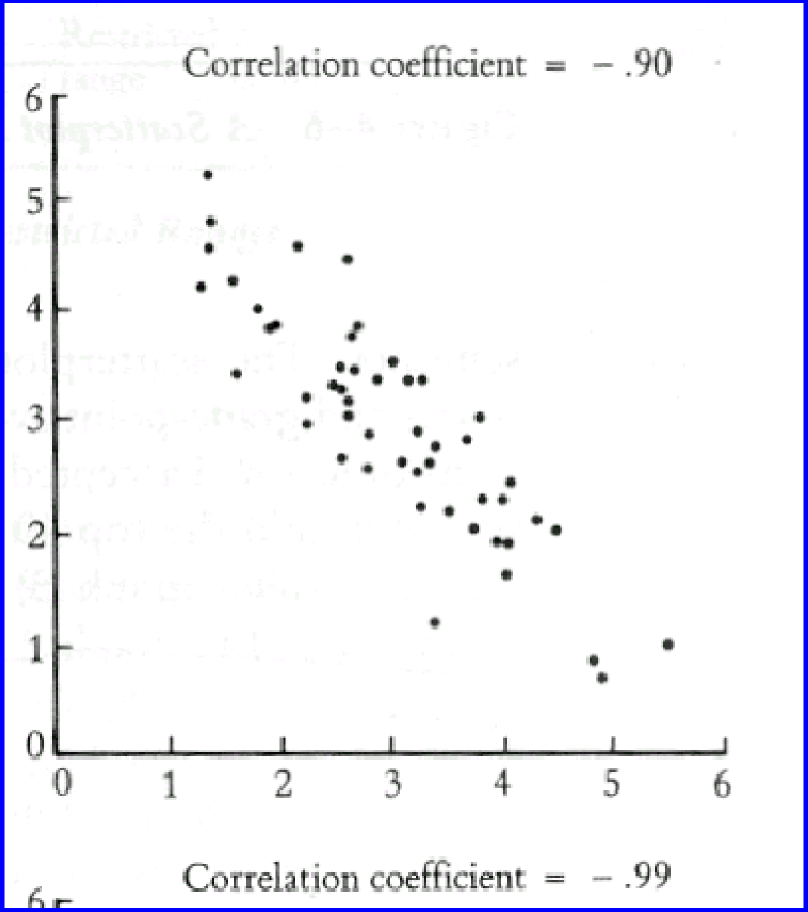
# of Malformed Cells in Lung Biopsy




# of Cigarettes Smoked per Day x 10

# Correlation

Lung Capacity

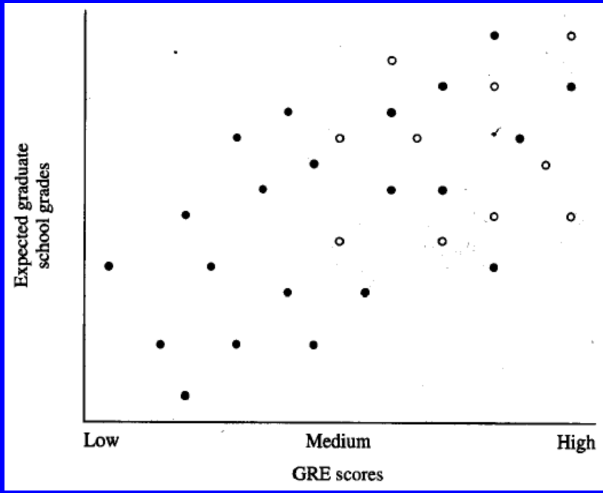



# of cigarettes smoked per day x 10

# Methodology: Restriction of Range

Restriction of Range cases an artificially low (underestimated) value of r.

- E.G. using just high GRE scores represented by the open circles.
- Common when using the scores to determine Who is used in the correlational analysis.
  - E.G.: Only applicants with high GRE scores get into Grad School.



# Computing r

Raw Scores

$$r = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{[\sum (X - \bar{X})^2][\sum (Y - \bar{Y})^2]}}$$

Deviation Scores

$$r = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}}$$

# Computing r, cont.

$$r = \frac{\Sigma(z_X z_Y)}{N}$$

Z-scores

# Can You Predict $Y_i$ If: You Know $X_i$?

$$r = \frac{\Sigma (X - \bar{X})(Y - \bar{Y})}{\sqrt{[\Sigma (X - \bar{X})^2][\Sigma (Y - \bar{Y})^2]}}$$

| X | $d_i$ | $d_{ix} * d_{ix}$ | | Y | $d_i$ | $d_{iy} * d_{iy}$ | | $d_{ix} * d_{iy}$ |
|---|---|---|---|---|---|---|---|---|
| 10 | 10 | 100 | | 10 | 10 | 100 | | 100 |
| 10 | 10 | 100 | | 10 | 10 | 100 | | 100 |
| -10 | -10 | 100 | | -10 | -10 | 100 | | 100 |
| -10 | -10 | 100 | | -10 | -10 | 100 | | 100 |
| **X-bar=0** | | | **Y-bar=** | **Y-bar=0** | | | | |
| | | **SUM** | | | | **SUM** | | **SUM** |
| | | **400** | | | | **400** | | **400** |

# Can You Predict $Y_i$ If: You Know $X_i$?

$$r = \frac{\Sigma (X - \bar{X})(Y - \bar{Y})}{\sqrt{[\Sigma (X - \bar{X})^2][\Sigma (Y - \bar{Y})^2]}}$$

| X | $d_i$ | $d_{ix} * d_{ix}$ | | Y | $d_i$ | $d_{iy} * d_{iy}$ | | $d_{ix} * d_{iy}$ |
|---|---|---|---|---|---|---|---|---|
| 10 | 10 | 100 | | 10 | 10 | 100 | | 100 |
| 10 | 10 | 100 | | -10 | -10 | 100 | | -100 |
| -10 | -10 | 100 | | 10 | 10 | 100 | | -100 |
| -10 | -10 | 100 | | -10 | -10 | 100 | | 100 |
| **X-bar=0** | | | **Y-bar=** | **Y-bar=0** | | | | |
| | | **SUM** | | | | **SUM** | | **SUM** |
| | | **400** | | | | **400** | | **0** |

# Methodology: Reliability

An instrument used to measure a Trait (vs. State) must be Reliable.

Measurements taken twice on the same subjects should agree.

Disagreement:

- Not a Trait
- Poor Instrument

Criterion for Reliability: r=0.80

Coefficient of Stability:

Correlation of measures taken more than 6mo. apart

# Regression

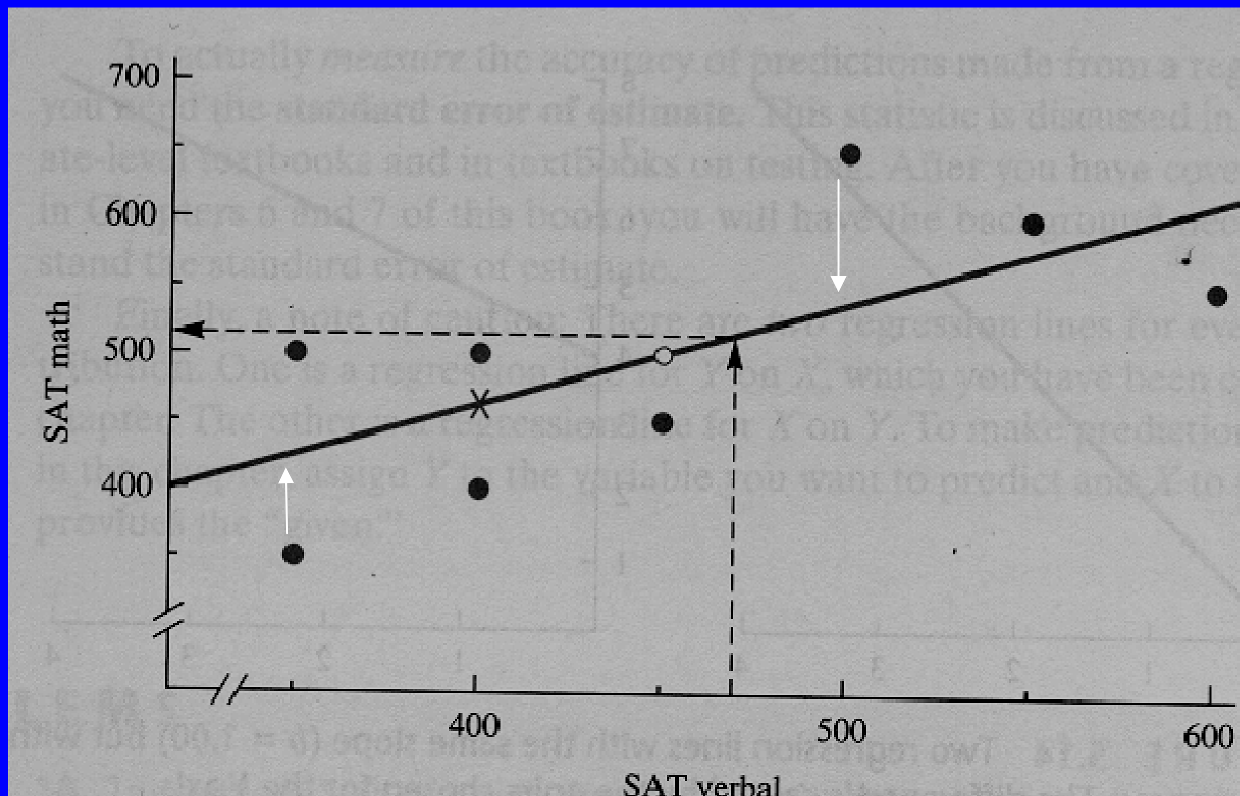
Creates a line of "Best Fit" running through the data

- Uses Method of Least Squares
  - The smallest Squared Distances between the Points and The Line

Y-hat = a +b*X and y= a +b*X-hat

- a=intercept b=slope
- The Regression Line (line of best fit) give you a & b
- Plug in X to predict Y, or Y to predict X

# Regression, cont.




Method of Lest Squares:

- Minimizes deviations from regression line
- Therefore, minimizes Errors of Prediction

# Regression, cont.

Correlation between X & Y = Correlation between Y & Y-hat

Error of Estimation: Difference between Y and Y-hat

Standard Error of Estimation: sqrt $(\Sigma(\text{Y-Y-hat})^2/n)$

- Remember what "Standard" means
- The higher the correlation:
  - The lower the Standard Error of Estimation

Shrinkage: Reduction in size of correlation between sample correlation and the population correlation which it measures

# Multiple Correlation & Regression

Using several measures to predict a measure or future measure

$\text{Y-hat} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$

- Y-hat is the Dependent Variable
- $X_1$, $X_2$, $X_3$, & $X_4$ are the Predictor (Independent) Variables

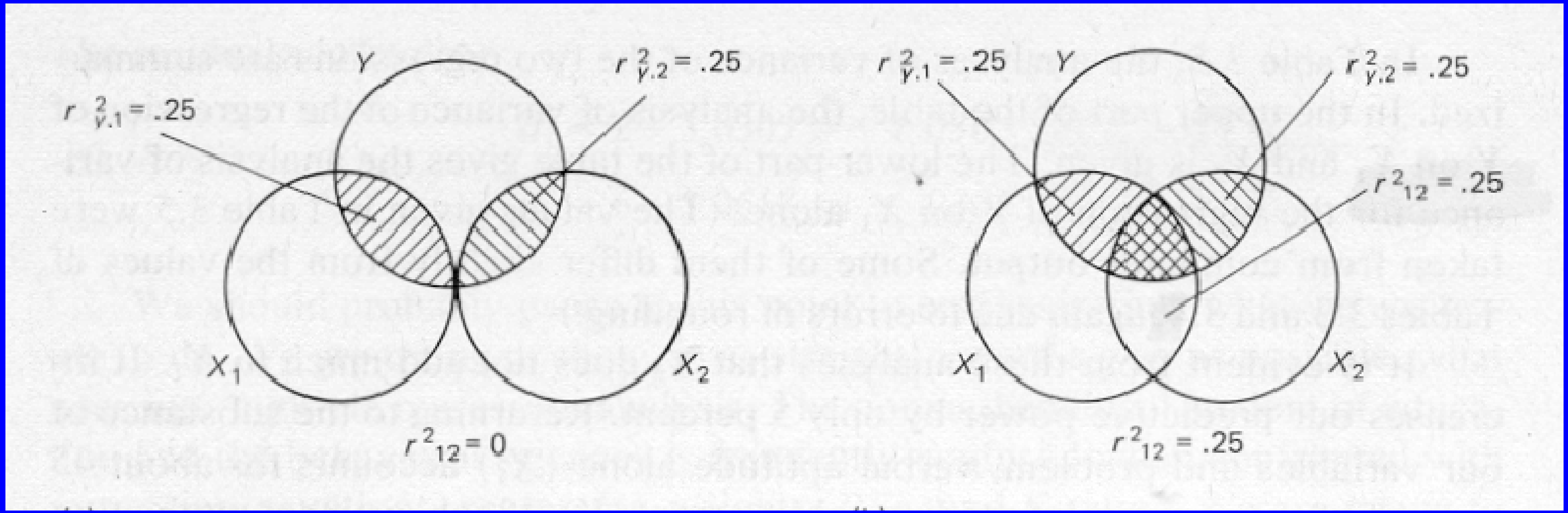
$\text{College GPA-hat} = a + b_1\text{H.S.GPA} + b_2\text{SAT} + b_3\text{ACT} + b_4\text{HoursWork}$

R = Multiple Correlation (Range: -1 - 0 - +1)

$^{R2}$ = Coefficient of Determination (R*R * 100; 0 - 100%)

Uses Partial Correlations for all but the first Predictor Variable

# Partial Correlations




The relationship (shared variance) between two variables when the variance which they BOTH share with a third variable is removed

Used in multiple regression to subtract Redundant variance when Assessing the Combined relationship between the Predictor Variables And the Dependent Variable. E.G., H.S. GPA and SAT scores.

# Step-wise Regression

Build your regression equation one dependent variable at a time.

- Start with the P.V. with the highest simple correlation with the DV

- Compute the partial correlations between the remaining PVs and
  The DV
  Take the PV with the highest partial correlation

- Compute the partial correlations between the remaining PVs and
  The DV with the redundancy with the First Two Pvs removed.
  Take the PV with the highest partial correlation.

- Keep going until you run out of PVs

# Step-wise Regression, cont.

Simple Correlations with college GPA:

| | |
|---|---|
| HS GPA | =.6 |
| SAT | =.5 |
| ACT | =.48 (but highly Redundant with SAT, measures same thing |
| Work | =-.3 |

College GPA-hat = a + $b_1$H.S.GPA + $b_2$SAT + $b_3$HoursWork + $b_4$ACT

# Stepwise Multiple Regression

DEPENDENT VARIABLE.. FRESHGPA

SUMMARY TABLE

| VARIABLE | MULTIPLE R | R SQUARE | RSQ CHANGE | SIMPLE R | B |
|---|---|---|---|---|---|
| COLBOARD | 0.70000 | 0.49000 | 0.49000 | 0.70000 | 0.00821 |
| HIGHSCH | 0.75820 | 0.57487 | 0.08487 | 0.01300 | -0.01819 |
| FAMINC | 0.76278 | 0.58183 | 0.00697 | 0.12000 | 0.01931 |
| (CONSTANT) | | | | | -1.35188 |

DEPENDENT VARIABLE.. INVINDEX INVESTORS INDEX 1949=100

SUMMARY TABLE

| VARIABLE | | MULTIPLE R | R SQUARE | RSQ CHANGE | SIMPLE R | B | BETA |
|---|---|---|---|---|---|---|---|
| GNP | GROSS NATIONAL PRODUCT | 0.93729 | 0.87852 | 0.87852 | 0.93729 | 0.01574 | 1.08714 |
| CORPPROF | CORPORATE PROFITS BEFORE TAXES | 0.95153 | 0.90540 | 0.02689 | 0.87912 | -0.15462 | -0.55669 |
| CORPDIV | CORPORATE DIVIDENDS PAID | 0.97774 | 0.95598 | 0.05058 | 0.93667 | 0.42586 | 0.45524 |
| (CONSTANT) | | | | | | -111.70268 | |

# Shrinkage

Step 1: Construct Regression Equation using sample which has already graduated from college.

Step 2: Use the a, b1, b2, b3, b3 from this equation to Predict College GPA (Y-hat) of high school graduates/applicants

The regression equation will do a better job of predicting College GPA (Y-hat) of the original sample because it factors in all the Idiosyncratic relationships (correlations) of the original sample.

Shrinkage: Original $R^2$ will be Larger than future $R^2$s

# Forced Order of Entry

Specify order in which PVs are added to the regression equation

Used to test (1) Hypotheses and to control for (2) Confounding Variables

E.G.: Is there gender bias in the salaries of lawyers?

- Point-Biserial Correlation ($r_{pb}$) of Gender and Salary: $r_{pb}$ =0.4
  Correlation between Dichotomous and Continuous Variable
- But females are younger, less experienced, & have fewer years on current job

1. Create Multiple Regression formula with all the other variables
2. Then Add the test variable (Gender)
3. Look at $R^2$: Does it Increase a lot or little at all?
   If $R^2$ goes up appreciably, then Gender has a Unique Influence

# Other Types Of Correlation

Pearson Product-Moment Correlation:

- Standard correlation
- r = Ratio of shared variance to total variance
- Requires two continuous variables of interval/ratio level

Point Biserial correlation ($r_{pbs}$ or $r_{pb}$):

- One Truly Dichotomous (only two values)
- One continuous (interval/ratio) variable
- Measures proportion of variance in the continuous variable
  - Which can be related to group membership
    - E.g., Biserial correlation between height and gender

# Discriminant Function Analysis
# Logistic Regression

Look at relationship between Group Membership (DV) and PVs
Using a regression equation.

Depression = a + $b_1$hours of sleep + $b_2$blood pressure
+ $b_3$calories consumed

Code Depression: 0 for No; 1 for Yes
If Y-hat >0.5, predict that subject has depression

Look at:
Sensitivity: Percent of Depressed individuals found
Selectivity: Percent of Positives which are Correct

# Discriminant Function Analysis
# Logistic Regression

Four possible outcomes for each prediction (Y-hat):

| | Y = 0 | Y = 1 |
|---|---|---|
| Y-hat < 0.5 | Correct Rejection | Miss |
| Y-hat > 0.5 | False Positive | Hit |

↑ Sensitivity
% Hits ↓

← Selectivity →
% Correct Hits

# Discriminant Function Analysis
# Logistic Regression

Expect Shrinkage:

Double Cross Validation:

1. Split sample in half
2. Construct Regression Equations for each
3. Use Regression Equations to predict Other Sample DV
   Look at Sensitivity and Selectivity
   If DV is continuous look at correlation between Y and Y-hat
   If IVs are valid predictors, both equations should be good
4. Construct New regression equation using combined samples

# Discriminant Function Analysis
# Logistic Regression

Can have more than two groups, if they are related quantitatively.

| | | |
|---|---|---|
| E.G.: | Mania | = 1 |
| | Normal | = 0 |
| | Depression | = -1 |

# Which Procedure To Use?

<u>Logistic Regression produces a more efficient regression equation Than does Discriminant Function Analysis</u>:

Greater Sensitivity and Selectivity