Genetic Algorithms

- Evolutionary computation
- Prototypical GA
- An example: GABIL
- Genetic Programming
- Individual learning and population evolution
Evolutionary Computation

1. Computational procedures patterned after biological evolution

2. Search procedure that probabilistically applies search operators to a set of points in the search space

• Also popular with optimization folks
Biological Evolution

Lamarck and others:
• Species “transmute” over time

Darwin and Wallace:
• Consistent, heritable variation among individuals in population
• Natural selection of the fittest

Mendel and genetics:
• A mechanism for inheriting traits
• Genotype \rightarrow \text{Phenotype} mapping
Genetic Algorithm

\( \text{GA}(\text{Fitness, FitnessThreshold, } p, r, m) \)

- **Initialize**: \( P \leftarrow p \) random hypotheses
- **Evaluate**: for each \( h \) in \( P \), compute \( \text{Fitness}(h) \)
- **While** \( \max_h \text{Fitness}(h) < \text{FitnessThreshold} \)
  1. **Select**: probabilistically select \((1-r)p\) members of \( P \) to add to \( P_s \)
     \[
     \Pr(h_i) = \frac{\text{Fitness}(h_i)}{\sum_{j=1}^{p} \text{Fitness}(h_j)}
     \]
  2. **Crossover**: probabilistically select pairs of hypotheses from \( P \).
     For each pair \(< h_1, h_2 >\), produce two offspring by applying the \text{Crossover} operator. Add all offspring to \( P_s \)
  3. **Mutate**: invert a randomly selected bit in \( mp \) random members of \( P_s \)
  4. **Update**: \( P \leftarrow P_s \)
  5. **Evaluate**: for each \( h \) in \( P \), compute \( \text{Fitness}(h) \)

Return the hypothesis from \( P \) that has the highest fitness
Representing Hypotheses

Represent

\[(Type = \text{Car} \lor \text{Minivan}) \land (\text{Tires} = \text{Blackwall})\]

by

\[
\begin{array}{ccc}
\text{Type} & \text{Tires} \\
011      & 10 \\
\end{array}
\]

Represent

IF \( (Type = \text{SUV}) \) THEN \( (\text{NiceCar} = \text{yes}) \)

by

\[
\begin{array}{ccc}
\text{Type} & \text{Tires} & \text{NiceCar} \\
100      & 11       & 10 \\
\end{array}
\]
Operators for Genetic Algorithms

**Parent Strings**

- **Single Point Crossover**
  - Parent 1: 101100101001
  - Parent 2: 000011100101
  - Offspring: 101111100101
  - Offspring: 000000101001

- **Two Point Crossover**
  - Parent 1: 101100101001
  - Parent 2: 000011100101
  - Offspring: 100111100001
  - Offspring: 001000101101

- **Uniform Crossover**
  - Parent 1: 101100101001
  - Parent 2: 000011100101
  - Offspring: 100111100001
  - Offspring: 001000101101

- **Point Mutation**
  - Parent: 101100101001
  - Offspring: 101100100001
Selecting Most Fit Hypothesis

Fitness proportionate selection:

\[ \Pr(h_i) = \frac{\text{Fitness}(h_i)}{\sum_{j=1}^{p} \text{Fitness}(h_j)} \]

... can lead to **crowding**

Tournament selection:

- Pick \( h_1, h_2 \) at random with uniform probability
- With probability \( p \), select the more fit

Rank selection:

- Sort all hypotheses by fitness
- Probability of selection is proportional to rank
GABIL (DeJong et al. 1993)

Learn disjunctive set of propositional rules, competitive with C4.5

**Fitness:**

\[ \text{Fitness}(h) = (\text{correct}(h))^2 \]

**Representation:**

\[
\text{IF } a_1 = \text{T} \land a_2 = \text{F} \text{ THEN } c = \text{T}; \text{ if } a_2 = \text{T} \text{ THEN } c = \text{F}
\]

represented by

\[
\begin{array}{ccc}
a_1 & a_2 & c \\
10 & 01 & 1 \\
11 & 10 & 0
\end{array}
\]

**Genetic operators:** ???

- want variable length rule sets
- want only well-formed bitstring hypotheses
Crossover with Variable-Length Bitstrings

Start with

\[
\begin{align*}
& a_1 \ a_2 \ c \quad a_1 \ a_2 \ c \\
& h_1 : 10 \ 01 \ 1 \quad 11 \ 10 \ 0 \\
& h_2 : 01 \ 11 \ 0 \quad 10 \ 01 \ 0 \\
\end{align*}
\]

1. Choose crossover points for \( h_1 \), e.g., after bits 1,8

\[
\begin{align*}
& h_1 : 1[0 \ 01 \ 1 \quad 11 \ 1]0 \ 0 \\
\end{align*}
\]

2. Now restrict points in \( h_2 \) to those that produce bitstrings with well-defined semantics, e.g., <1,3>, <1,8>, <6,8>

If we choose <1,3>:

\[
\begin{align*}
& h_2 : 0[1 \ 1]1 \ 0 \quad 10 \ 01 \ 0 \\
\end{align*}
\]

Result is:

\[
\begin{align*}
& a_1 \ a_2 \ c \quad a_1 \ a_2 \ c \quad a_1 \ a_2 \ c \\
& h_3 : 11 \ 10 \ 0 \\
& h_4 : 00 \ 01 \ 1 \quad 11 \ 11 \ 0 \quad 10 \ 01 \ 0 \\
\end{align*}
\]
GABIL Extensions

Add new genetic operators, applied probabilistically

1. *AddAlternative*: generalize constraint on \( a_i \) by changing a 0 to 1

2. *DropCondition*: generalize constraint on \( a_i \) by changing every 0 to 1

And, add new field to bit string to determine whether to allow these:

\[
\begin{array}{ccc}
  a_1 & a_2 & c & a_1 & a_2 & c & AA & DC \\
  10 & 01 & 1 & 11 & 10 & 0 & 1 & 0
\end{array}
\]

So now the learning strategy also evolves!
GABIL Results

Performance of GABIL comparable to symbolic rule/tree learning methods C4.5, ID5R, AQ14

Average performance on a set of 12 synthetic problems:

- GABIL without AA and DC operators: 92.1% accuracy
- GABIL with AA and DC operators: 95.2% accuracy
- Symbolic learning methods ranged from 91.2% to 96.6% accuracy
Schemas

How to characterize evolution of population in GA?

*Schema* = string containing 0, 1, * (“don’t care”)

- Typical schema: 10**0*
- Instances of above schema: 101101, 100000, ...

Characterize population by number of instances representing each possible schema

- \( m(s, t) \) = number of instances of schema \( s \) in population at time \( t \)
Consider Just Selection

- $f(t) =$ average fitness of population at time $t$
- $m(s,t) =$ instances of schema $s$ in population at time $t$
- $\hat{u}(s,t) =$ average fitness of instances of $s$ at time $t$

Probability of selecting $h$ in one selection step

$$Pr(h) = \frac{\sum_{i=1}^{n} f(h_i)}{nf(t)} = \frac{f(h)}{nf(t)}$$

Probability of selecting an instances of $s$ in one step

$$Pr(h \in s) = \sum_{h \in s \cap p_t} \frac{f(h)}{nf(t)} = \frac{\hat{u}(s,t)}{nf(t)} m(s,t)$$

Expected number of instances of $s$ after $n$ selections

$$E[m(s,t + 1)] = \frac{\hat{u}(s,t)}{f(t)} m(s,t)$$
Schema Theorem

\[ E[m(s,t + 1)] \geq \frac{\hat{u}(s,t)}{f(t)} m(s,t) \left( 1 - p_c \frac{d(s)}{l - 1} \right) (1 - p_m)^{o(s)} \]

- \( m(s,t) \) = instances of schema \( s \) in population at time \( t \)
- \( \bar{f}(t) \) = average fitness of population at time \( t \)
- \( \hat{u}(s,t) \) = average fitness of instances of \( s \) at time \( t \)
- \( p_c \) = probability of single point crossover operator
- \( p_m \) = probability of mutation operator
- \( l \) = length of single bit strings
- \( o(s) \) = number of defined (non "*" ) bits in \( s \)
- \( d(s) \) = distance between left-, right - most defined bits in \( s \)
Genetic Programming

Population of programs represented by trees

Example:

\[ \sin(x) + \sqrt{x^2 + y} \]
Crossover
Block Problem

Goal: spell UNIVERSAL

Terminals:
- CS ("current stack") = name of top block on stack, or False
- TB ("top correct block") = name of topmost correct block on stack
- NN ("next necessary") = name of next block needed above TB in the stack
Block Problem Primitives

Primitive functions:

- `(MS x)`: ("move to stack"), if block `x` is on the table, moves `x` to the top of the stack and returns True. Otherwise, does nothing and returns False.
- `(MT x)`: ("move to table"), if block `x` is somewhere in the stack, moves the block at the top of the stack to the table and returns True. Otherwise, returns False.
- `(EQ x y)`: ("equal"), returns True if `x` equals `y`, False otherwise.
- `(NOT x)`: returns True if `x` = False, else return False.
- `(DU x y)`: ("do until") executes the expression `x` repeatedly until expression `y` returns the value True.
Learned Program

Trained to fit 166 test problems

Using population of 300 programs, found this after 10 generations:

\[
(EQ \ (DU \ (MT \ CS) \ (NOT \ CS)) \)
(DU \ (MS \ NN) \ (NOT \ NN))
\]
Genetic Programming

More interesting example: design electronic filter circuits

- Individuals are programs that transform the beginning circuit to a final circuit by adding/subtracting components and connections
- Use population of 640,000, run on 64 node parallel process
- Discovers circuits competitive with best human designs
GP for Classifying Images

**Fitness:** based on coverage and accuracy

**Representation:**
- Primitives include Add, Sub, Mult, Div, Not, Max, Min, Read, Write, If-Then-Else, Either, Pixel, Least, Most, Ave, Variance, Difference, Mini, Library
- Mini refers to a local subroutine that is separately co-evolved
- Library refers to a global library subroutine (evolved by selecting the most useful minis)

**Genetic operators:**
- Crossover, mutation
- Create “mating pools” and use rank proportionate reproduction
Biological Evolution

Lamarck (19th century)

- Believed individual genetic makeup was altered by lifetime experience
- Current evidence contradicts this view

What is the impact of individual learning on population evolution?
Baldwin Effect

Assume

- Individual learning has no direct influence on individual DNA
- But ability to learn reduces the need to “hard wire” traits in DNA

Then

- Ability of individuals to learn will support more diverse gene pool
  - Because learning allows individuals with various “hard wired” traits to be successful
- More diverse gene pool will support faster evolution of gene pool

→ individual learning (indirectly) increases rate of evolution
Baldwin Effect (Example)

Plausible example:
1. New predator appears in environment
2. Individuals who can learn (to avoid it) will be selected
3. Increase in learning individuals will support more diverse gene pool
4. Resulting in faster evolution
5. Possibly resulting in new non-learned traits such as instinctive fear of predator
Computer Experiments on Baldwin Effect

Evolve simple neural networks:
- Some network weights fixed during lifetime, others trainable
- Genetic makeup determines which are fixed, and their weight values

Results:
- With no individual learning, population failed to improve over time
- When individual learning allowed
  - Early generations: population contained many individuals with many trainable weights
  - Later generations: higher fitness, white number of trainable weights decreased
Bucket Brigade

- Evaluation of fitness can be very indirect
  - consider learning rule set for multi-step decision making
  - bucket brigade algorithm:
    - rule leading to goal receives reward
    - that rule turns around and contributes some of its reward to its predecessor
  - no issue of assigning credit/blame to individual steps
Evolutionary Programming Summary

• Approach learning as optimization problem (optimizes fitness)

• Concepts as chromosomes
  – representation issues:
    • near values should have near chromosomes (grey coding)
    • variable length encodings (GABIL, Genetic Programming)

• Genetic algorithm (GA) basics
  – population
  – fitness function
  – fitness proportionate reproduction
Evolutionary Programming Summary

• Genetic algorithm (GA) basics
  – reproduction operators
    • crossover
    • single, multi, uniform
    • mutation
    • application specific operators

• Genetic Programming (GP)
  – programs as trees
  – genetic operations applied to pairs of trees
Evolutionary Programming Summary

• Other evolution issues
  – adaptation of chromosome during lifetime (Lamarck)
  – Baldwin effect (ability to learn indirectly supports better populations)

• Schema theorem
  – good ideas are schemas (some features set, others *)
  – over time good schemas are concentrated in population