Learning Sets of Rules

- Sequential covering algorithms
- FOIL
- Induction as the inverse of deduction
- Inductive Logic Programming
Learning Disjunctive Sets of Rules

Method 1: Learn decision tree, convert to rules
Method 2: Sequential covering algorithm
1. Learn one rule with high accuracy, any coverage
2. Remove positive examples covered by this rule
3. Repeat
SequentialCoveringAlgorithm

SEQUENTIAL-COVERING(Target_attr,Attrs,Examples,Thresh)

Learned_rules ← \{

Rule ← LEARN-ONE-RULE(Target_attr,Attrs,Examples)

while PERFORMANCE(Rule,Examples) > Thresh do

- Learned_rules ← Learned_rules + Rule

- Examples ← Examples - \{examples correctly classified by Rule\}

- Rule ← LEARN-ONE-RULE(Target_attr,Attrs,Examples)

Learned_rules ← sort Learned_rules according to

PERFORMANCE over Examples

return Learned_rules
Learn-One-Rule

IF
THEN CoolCar=Yes

IF Type = SUV
THEN CoolCar=Yes

IF Type = Car
THEN CoolCar=Yes

IF Type = SUV AND Doors = 2
THEN CoolCar=Yes

IF Type = SUV AND Doors = 4
THEN CoolCar=Yes

IF Type = SUV AND Color = Red
THEN CoolCar=Yes
Covering Rules

\( Pos \leftarrow \text{positive Examples} \)
\( Neg \leftarrow \text{negative Examples} \)

while \( Pos \) do (\textit{Learn a New Rule})

\( NewRule \leftarrow \text{most general rule possible} \)
\( NegExamplesCovered \leftarrow Neg \)

while \( NegExamplesCovered \) do

    Add a new literal to specialize \( NewRule \)
    1. \( \text{Candidate_literals} \leftarrow \text{generate candidates} \)
    2. \( \text{Best_literal} \leftarrow \arg\max_{L \in \text{candidate_literals}} \text{PERFORMANCE}(\text{SPECIALIZE-RULE}(NewRule, L)) \)
    3. Add \( \text{Best_literal} \) to \( NewRule \) preconditions
    4. \( NegExamplesCovered \leftarrow \text{subset of } NegExamplesCovered \text{ that satisfies } NewRule \text{ preconditions} \)

\( Learned\_rules \leftarrow Learned\_rules + NewRule \)
\( Pos \leftarrow Pos - \{\text{members of } Pos \text{ covered by } NewRule\} \)

Return \( Learned\_rules \)
Subtleties: Learning One Rule

1. May use *beam search*
2. Easily generalize to multi-valued target functions
3. Choose evaluation function to guide search:
   - Entropy (i.e., information gain)
   - Sample accuracy: \( \frac{n_c}{n} \)
     where \( n_c \) = correct predictions, \( n = \) all predictions
   - m estimate: \( \frac{n_c + mp}{n + m} \)
Variants of Rule Learning Programs

• *Sequential* or *simultaneous* covering of data?
• General $\rightarrow$ specific, or specific $\rightarrow$ general?
• Generate-and-test, or example-driven?
• Whether and how to post-prune?
• What statistical evaluation functions?
Learning First Order Rules

Why do that?

• Can learn sets of rules such as

  \[ \text{Ancestor}(x,y) \leftarrow \text{Parent}(x,y) \]

  \[ \text{Ancestor}(x,y) \leftarrow \text{Parent}(x,z) \land \text{Ancestor}(z,y) \]

• General purpose programming language

  PROLOG: programs are sets of such rules
First Order Rule for Classifying Web Pages

From (Slattery, 1997)

course(A) ←
    has-word(A,instructor),
    NOT has-word(A,good),
    link-from(A,B)
    has-word(B,assignment),
    NOT link-from(B,C)

Train: 31/31, Test 31/34
FOIL

FOIL(Target_predicate, Predicates, Examples)

Pos ← positive Examples
Neg ← negative Examples

while Pos do (Learn a New Rule)

NewRule ← most general rule possible
NegExamplesCovered ← Neg

while NegExamplesCovered do
    Add a new literal to specialize NewRule
    1. Candidate_literals ← generate candidates
    2. Best_literal ← argmax_{L ∈ candidate_literal} FOIL_GAIN(L, NewRule)
    3. Add Best_literal to NewRule preconditions
    4. NegExamplesCovered ← subset of NegExamplesCovered that satisfies NewRule preconditions

Learned_rules ← Learned_rules + NewRule
Pos ← Pos - {members of Pos covered by NewRule}

Return Learned_rules
Specializing Rules in FOIL

Learning rule: \( P(x_1, x_2, \ldots, x_k) \leftarrow L_1 \ldots L_n \)

Candidate specializations add new literal of form:

- \( Q(v_1, \ldots, v_r) \), where at least one of the \( v_i \) in the created literal must already exist as a variable in the rule

- \( Equal(x_j, x_k) \), where \( x_j \) and \( x_k \) are variables already present in the rule

- The negation of either of the above forms of literals
Information Gain in FOIL

\[
FOIL\_GAIN(L, R) \equiv t \left( \log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0} \right)
\]

Where

- \( L \) is the candidate literal to add to rule \( R \)
- \( p_0 \) = number of positive bindings of \( R \)
- \( n_0 \) = number of negative bindings of \( R \)
- \( p_1 \) = number of positive bindings of \( R+L \)
- \( n_1 \) = number of negative bindings of \( R+L \)
- \( t \) is the number of positive bindings of \( R \) also covered by \( R+L \)

Note

- \(-\log_2 \frac{p_0}{p_0 + n_0}\) is optimal number of bits to indicate the class of a positive binding covered by \( R \)
Induction as Inverted Deduction

Induction is finding $h$ such that

$$(\forall <x_i, f(x_i)> \in D) \ B \land h \land x_i \vdash \neg f(x_i)$$

where

- $x_i$ is the $i$th training instance
- $f(x_i)$ is the target function value for $x_i$
- $B$ is other background knowledge

So let’s design inductive algorithms by inverting operators for automated deduction!
Induction as Inverted Deduction

“pairs of people, \(<u,v>\) such that child of \(u\) is \(v\),”

\[ f(x_i) : Child(Bob,Sharon) \]
\[ x_i : Male(Bob),Female(Sharon),Father(Sharon,Bob) \]
\[ B : Parent(u,v) \leftarrow Father(u,v) \]

What satisfies \((\forall <x_i,f(x_i)> \in D) B \land h \land x_i \mid \neg f(x_i)\)?

\[ h_1 : Child(u,v) \leftarrow Father(v,u) \]
\[ h_2 : Child(u,v) \leftarrow Parent(v,u) \]
Induction and Deduction

Induction is, in fact, the inverse operation of deduction, and cannot be conceived to exist without the corresponding operation, so that the question of relative importance cannot arise. Who thinks of asking whether addition or subtraction is the more important process in arithmetic? But at the same time much difference in difficulty may exist between a direct and inverse operation; ... it must be allowed that inductive investigations are of a far higher degree of difficulty and complexity than any question of deduction ... (Jevons, 1874)
Induction as Inverted Deduction

We have mechanical deductive operators

\[ F(A,B) = C, \text{ where } A \land B \vdash C \]

need inductive operators

\[ O(B,D) = h \text{ where } \]

\[ (\forall \langle x_i, f(x_i) \rangle \in D) B \land h \land x_i \vdash f(x_i) \]
Induction as Inverted Deduction

Positives:
- Subsumes earlier idea of finding $h$ that “fits” training data
- Domain theory $B$ helps define meaning of “fit” the data
  \[ B \land h \land x_i \models f(x_i) \]
- Suggests algorithms that search $H$ guided by $B$

Negatives:
- Doesn’t allow for noisy data. Consider
  \[ (\forall <x_i, f(x_i)> \in D) B \land h \land x_i \models \neg f(x_i) \]
- First order logic gives a huge hypothesis space $H$
  - overfitting…
  - intractability of calculating all acceptable $h$’s
Deduction: Resolution Rule

\[
P \lor L \\
\neg L \lor R \\
\hline \\
P \lor R
\]

1. Given initial clauses \(C1\) and \(C2\), find a literal \(L\) from clause \(C1\) such that \(\neg L\) occurs in clause \(C2\).

2. Form the resolvent \(C\) by including all literals from \(C1\) and \(C2\), except for \(L\) and \(\neg L\). More precisely, the set of literals occurring in the conclusion \(C\) is

\[
C = (C1 - \{L\}) \cup (C2 - \{\neg L\})
\]

where \(\cup\) denotes set union, and “-” set difference.
Inverting Resolution

\[ C_1: \text{PassExam} \lor \neg\text{KnowMaterial} \quad C_2: \text{KnowMaterial} \lor \neg\text{Study} \]

\[ C: \text{PassExam} \lor \neg\text{Study} \]
Inverted Resolution (Propositional)

1. Given initial clauses $C_1$ and $C$, find a literal $L$ that occurs in clause $C_1$, but not in clause $C$.

2. Form the second clause $C_2$ by including the following literals

$$C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\}$$
First Order Resolution

1. Find a literal $L_1$ from clause $C_1$, literal $L_2$ from clause $C_2$, and substitution $\theta$ such that

$$L_1 \theta = \neg L_2 \theta$$

2. Form the resolvent $C$ by including all literals from $C_1 \theta$ and $C_2 \theta$, except for $L_1 \theta$ and $\neg L_2 \theta$. More precisely, the set of literals occurring in the conclusion is

$$C = (C_1 - \{L_1\}) \theta \cup (C_2 - \{L_2\}) \theta$$

Inverting:

$$C_2 = (C - (C_1 - \{L_1\}) \theta_1) \theta_2^{-1} \cup \{\neg L_1 \theta_1 \theta_2^{-1}\}$$
Cigol

\[
\begin{align*}
\text{Father}(\text{Bob}, \text{Shannon}) & \quad \text{GrandChild}(y, x) \lor \neg \text{Father}(x, z) \lor \neg \text{Father}(z, y)) \\
\text{Father}(\text{Bob}, \text{Y}) & \quad \text{GrandChild}(\text{Bob}, x) \lor \neg \text{Father}(x, \text{Bob}) \\
\{\text{Shannon}/x\} & \quad \{\text{Bob}/y, \text{Tom}/z\}
\end{align*}
\]
Progol

PROGOL: Reduce combinatorial explosion by generating the most specific acceptable $h$

1. User specifies $H$ by stating predicates, functions, and forms of arguments allowed for each

2. PROGOL uses sequential covering algorithm.

   For each $\langle x_i, f(x_i) \rangle$
   - Find most specific hypothesis $h_i$ s.t.
     \[
     B \wedge h_i \wedge x_i \models f(x_i)
     \]
     actually, only considers k-step entailment

3. Conduct general-to-specific search bounded by specific hypothesis $h_i$, choosing hypothesis with minimum description length
Learning Rules Summary

• Rules: easy to understand
  – Sequential covering algorithm
  – generate one rule at a time
  – general to specific - add antecedents
  – specific to general - delete antecedents
  – Q: how to evaluate/stop?

• First order logic and covering
  – how to connect variables
  – FOIL
Learning Rules Summary (cont)

• Induction as inverted deduction
  – what background rule would allow deduction?
  – resolution
  – inverting resolution
  – and first order logic

• Cigol, Progol