**Action Refinement in Reinforcement Learning by Probability Smoothing**

By Thomas G. Dietterich & Didac Busquets
Speaker: Kai Xu

**Presentation Overview**
- **Background**
- The Probability Smoothing Method
- Experimental Study of Action Refinement
- Conclusion

**Background -- Model Based Reinforcement Learning (MBRL)**
- Experience gained during exploring is employed to learn the models of the state-action transition function and the reward function
- From the learned model, the optimal policy can be computed by many good algorithms
- MBRL is appropriate when the state and action space and relatively small and finite, and each exploring action is expensive.

**Background -- Methods To Reduce the Need For Training Data**
- By incorporate some kind of prior knowledge
- Previous Study: Abstraction knowledge across the states
  - So that the RL can generalize across states
- Abstraction knowledge across the actions (in this paper)
  - The RL assumes similar actions will have similar transition effects and rewards

**Background -- Action Refinement**
- Recall how human learns
  - Bad Kongfu Masters teach the students all the tricks at the beginning.
  - The students have to spend a long time to grasp all of them
- Good Kongfu Masters teach the students only the basic actions at the beginning.
- After the students grasp the basic skills, he teach them the subtleties among different similar actions.
- The students grasp all the tricks in a much shorter time.

**Action Refinement**
Action Refinement

* An RL algorithm initially treats a set of similar actions as a single abstraction
* Later, refines that abstraction action into individual actions.

The Probability Smoothing Method

* Background
* The Probability Smoothing Method
* Experimental Study of Action Refinement
* Conclusion

The Probability Smoothing Model

* Context:
  - The agent is interacting with an unknown but observable Markovian environment.
  - The environment contains a finite state set S, and a finite action set A.
  - The programmer groups set A into L disjoint action sets $A_1, A_2, ..., A_L$. Actions in the same subsets are 'similar'.

Let $N(s, a)$ denote the number of times action $a$ has been executed in state $s$. Let $N(s, a, s')$ denote the number of times this results in a transition to state $s'$. Let $W(s, a, s')$ denote the total rewards received when a transition from $s$ to $s'$ caused action $a$. Define the probability smoothing model $M$, such that

$$ P(s'|s,a) = \frac{\sum_{s', a'} \lambda^* N(s, a', s')}{\sum_{s', a'} \lambda^* N(s, a')} $$

$$ R(s, a, s') = \frac{\sum_{s', a'} \lambda^* W(s, a', s')}{\sum_{s', a'} \lambda^* N(s, a')} $$

Determine Smoothing Parameter $\lambda$

* Suppose the true transition probability from $s$ to $s'$ after executing action $a$ is $P(s'|s,a)$, and the estimate to this probability is $P(s'|s,a)$. We want to find a proper $\lambda$ such that $P(s'|s,a)$ would be a consistent estimator for the true probability.
* To determine which estimator is more appropriate, we need to define the error measure as the following:

$$ J(s, a) = \sum (P(s'|s,a) - P(s'|s,a))^2 $$

* So the problem is to find $\lambda$ which minimizes $J(s,a)$

Derivation of Optimal Smoothing Parameters in the simplest case

* Let's suppose there are only two similar actions, $a_1$ and $a_2$. The current state is $s$.
  - There are only two possible resulting states, $s'$ and $s''$.
  - Action $a_1$ has been applied on state $s$ for $N_1$ times. For $H_1$ times it transit to state $s'$.
  - Action $a_2$ has been applied on state $s$ for $N_2$ times. For $H_2$ times it transit to state $s'$.
Derivation of Optimal Smoothing Parameters in the simplest case

- Suppose the true transition probability from s to s' after exe a_i is p_{i}.
- Although H1/N1 is an estimator for p_i, it requires large number of trials.
- So we should use the smoothing model:
  \[ \hat{p}_i = \frac{H_1 + \lambda H_2}{N_1 + \lambda N_2} \]

Derivation of Optimal Smoothing Parameters in the simplest case

- After calculation, we find the most appropriate smoothing parameter
  \[ \lambda = \frac{V_1}{N^2 + V^2} \]
  where
  \[ V_1 = p_i(1 - p_i), \quad V_2 = p_j(1 - p_j) \]
  \[ \varepsilon = |p_i - p_j| \]
- Properties of using this \( \lambda \):
  \[ \lim_{N_1 \to \infty} \hat{p}_i = p_i \quad \text{and} \quad \lim_{N_2 \to \infty} \hat{p}_i = p_i \]

Determine the Level of Smoothing in Practice

- Therefore, the probability smoothing will converge to the optimal policy.
- This model can be expand to cases such as:
  - there are more than 2 similar actions
  - there are more than 2 possible resulting states
- We can use the resulting \( \lambda \) to build good estimator for the reward.

Determine the Level of Smoothing in Practice

- Big problem: In most practical cases, we will never know the true value for p_i, p_j, or \( \varepsilon \).
- A naive approach for choosing \( \lambda \) would be estimate \( p_i \) by H1/N1, estimate \( p_j \) by H2/N2
- But when the trial number is small, the variance to these estimates are very high. The result is poor.
- So the paper proposed to use "default smoothing", in which we assume the default values of \( p_i, p_j, \) and \( \varepsilon \), and plug in the value of N2 from the real data.

Determine the Level of Smoothing in Practice

- The author proposed to use default values
  \[ p_i = 0.1, \quad p_j = 0.15, \quad \varepsilon = |p_i - p_j| = 0.05 \]
  for the simplest case.
- They work well when \( \varepsilon < 0.15 \) for all values of \( p_i \).
- For cases that there are more than 2 possible resulting states, the author proposed to use default values
  \[ V_1 = 0.09, \quad V_2 = 0.1275, \quad \varepsilon = 0.0025 \]
Experimental Study of Action Refinement

**Background**
- The Probability Smoothing Method
- Experimental Study of Action Refinement
- Conclusion

**Experimental Study of Action Refinement -- Context**
- A toy maze with 81 non-terminal states and 2 terminal states.
- 16 actions from the cross-product of the 4 compass directions with 4 modifiers.
- Actions are grouped into 4 sets.
- To measure the performance of a policy, we compute the value function $V$ and sum the value of all 81 non-terminal states.
- The optimal policy has total value of 43.37

**Experimental Study of Action Refinement -- Compare With No Smoothing Method**
- Comparison of
  - probability smoothing, and
  - no smoothing ($\lambda = 0$)
- The probability smoothing model is much better

**Experimental Study of Action Refinement -- Compare with fixed smoothing & four-action**
- Comparison of
  - fixed smoothing ($\lambda = 1$)
  - four-action method
  - probability smoothing
- After 9.3 exploration steps, four-action method and probability smoothing method beat fixed smoothing.
- After 23 steps, probability smoothing method wins.

**Experimental Study of Action Refinement -- Conclusion**
- Conclusion for the previous experiment
  - The probability smoothing method is vastly superior to no-smoothing method.
  - If large training set is available, probability smoothing method is better than fix-smoothing and four-action method.

**Experimental Study of Action Refinement -- Sensitivity To The Size of Action Sets**
- Vary the # of action sets from 1, 2, 4, 8, and 16
- With similar actions be grouped together to the extent possible.
- 16 separate action sets gives high variance.
- One single action set gives high bias.
- 4 sets and 2 sets gave the best performance during the early part of the curve.
Experimental Study of Action Refinement -- Sensitivity To The Action Set Correctness

- At intermediate sample size (4), even random groupings give better performance than no smoothing.
- At large sample sizes, the bias in the random and bad groupings leads to worse performance than either no smoothing or well-chosen action sets.

Conclusions

- Probability Smoothing Method is introduced to action refinement to speed up RL applications.
- It significantly eases the designing of a set of good actions in RL.
- Probability smoothing parameter is determined by "default smoothing" and the corresponding # of trials.
- Good prior action set partition is critical to the performance.
Action Refinement in Reinforcement Learning by Probability Smoothing

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Comments by: Sameer Apte

Action Refinement applied to the Robot Navigation Problem

- Robot navigates by finding visual landmarks
- Robot’s camera has a viewing angle of 60 degrees
- The space around the robot was partitioned into six 60-degree sectors

Action Refinement applied to the Robot Navigation Problem

- An action called “Move While looking for Landmarks (MLL)” was defined
- Robot moves forward while aiming its camera in one of the six sectors to search for new visual landmarks
- Can define six MLL actions, one for each sector and let robot decide which sector to examine with the camera

Action Refinement applied to the Robot Navigation Problem

- Designers do not know which of these actions would be most useful
- Include all these actions in the MDP and let RL system determine which actions are useful
  - Problem: Large amount of exploration required to learn a good policy
- Train the robot several times, each time with a different set of actions
  - Problem: Even more training experiences required

Action Refinement applied to the Robot Navigation Problem

- Solution: Action Refinement
- We know that different variants of the MLL action have similar behavior
- Initially treat these similar actions as a single abstract action
- Later allow the learning algorithm to refine abstract action into individual actions
Direct vs. Model-Based Reinforcement Learning

-- Commentary on Kai Xu’s presentation
-- Commented by Ruinan Lu

Problem for comparison of the two approaches: single pendulum swing-up

- Make it swing!

Model-based RL

- Known reward function:
  \[ r(\theta, \tau) = ((\theta - \theta_d)^2 + \tau^2)\Delta \]
  - \( \theta \): angle of the pendulum
  - \( \theta_d \): desired angle for the inverted vertical state
  - \( \tau \): motor torque
  - \( \Delta \): time step

- Equation of motion: \( \theta^* = \tau - 9.81 \cos(\theta) \)

Q-learning

\[ Q(x_k, u_k) = Q(x_{k+1}, u_{k+1}) + \alpha[r(x_k, u_k) + \gamma Q(x_{k+1}, u_{k+1}) - Q(x_k, u_k)] + \epsilon(x_k, u_k) \]

- \( \alpha \): learning rate
- \( \gamma \): discount factor
- \( x \): state vector
- \( u \): control vector

- Optimal action: \( \arg\min_u Q(x_k, u) \)

Criteria

- Data efficiency
- Computing efficiency

Results
Conclusions

• Simple Dynamics favor MRL
  – Exploratory action is expensive
  – Exploration is performed on a physical system

• Cases favor Direct RL
  – More training experiences
  – Learner interacts with an inexpensive simulator