Example: Suppose, for a 3 minute egg timer, $n = 10$ and $\alpha = 0.05$

**Two-sided Alternative**

$H_0: \mu = 180$ seconds  
$H_a: \mu \neq 180$

Reject $H_0$ if $|t| = \left| \frac{\bar{y} - 180}{s/\sqrt{n}} \right| > 2.26$

Suppose $\bar{y} = 182.1$  
$s = 3.2$  
$n = 10$  
$SE_{\bar{y}} = \frac{3.2}{\sqrt{10}} = 1.01$

$t = \frac{182.1 - 180}{1.01} = 2.08$

Do not reject $H_0: \mu = 180$ for $\alpha = 0.05$ since $|t| < 2.26$

Reject $H_0: \mu \neq 180$ for $\alpha = 0.10$ since $|t| > 1.83$
\[2 \times 0.025 < \text{p-value} < 2 \times 0.050\]
\[0.05 < \text{p-value} < 0.10\]

- Reject \(H_0\) if p-value < \(\alpha\).
  - If \(\alpha = 0.05\), don’t reject \(H_0\).
  - If \(\alpha = 0.10\), do reject \(H_0\).
One-sided Alternative

$H_0: \mu \leq 180$ seconds

$H_a: \mu > 180$

Suppose $n = 10$ and $\alpha = 0.05$

$\bar{y} = 182.1 \quad s = 3.2 \quad n = 10 \quad SE_{\bar{y}} = \frac{3.2}{\sqrt{10}} = 1.01$

\[
t = \frac{\bar{y} - 180}{SE_{\bar{y}}} = \frac{182.1 - 180}{1.01} = 2.08
\]

- For $\alpha = 0.05$
  - Reject $H_0$ if $t > 1.833$ from t-table $t_{0.05}$
  - Since $t = 2.08 > 1.833$, reject $H_0$ if $\alpha = 0.05$.

- If $\alpha = 0.025$
  - Reject $H_0$ if $t > 2.262$ from t-table $t_{0.05}$
  - Since $t = 2.08 < 2.262$, do not reject $H_0$ if $\alpha = 0.025$.

$0.025 < p - \text{value} < 0.05$

If $y$ and $t$ are on the reject side of $H_0$ as in this example

1-sided p-value $= \frac{1}{2} \times$ 2-sided p-value
If $\bar{y}$ and $t$ are so far from $H_a$ that $\bar{y}$ on the other side of $H_0$ from $H_a$:

1-sided p-value = $1 - \frac{1}{2} \times$ 2-sided p-value

For example

$H_0: \mu \geq 180$ seconds

$H_a: \mu < 180$

$n = 10$ and $\alpha = 0.05$

$\bar{y} = 182.1 \quad s = 3.2 \quad n = 10 \quad SE_{\bar{y}} = \frac{3.2}{\sqrt{10}} = 1.01$

$$t = \frac{\bar{y} - 180}{SE_{\bar{y}}}$$

$$t = \frac{182.1 - 180}{1.01} = 2.08$$

- $\bar{y}$ is on the $H_0$ side of 180.
- $t$ needs to be negative to reject $H_0$
- Here $t > 0$ is far from the reject side of zero
  - Nowhere close to rejecting $H_0$
  - P-value is big

1-sided p-value = $1 - \frac{1}{2} \times$ (2-sided p-value)

$0.95 < p$-value $< 0.975$

Since $p$ is big, do not reject $H_0$. 
In general rejection regions are given by:

\[ H_0 : \mu = 180 \]

Reject \( H_0 \) if \(|t| > t_{\alpha/2} \)

\[ H_a : \mu \neq 180 \]

\[ H_a : \mu > 180 \] \hspace{1cm} Reject \( H_0 \) if \( t > t_{\alpha} \)

\[ H_a : \mu < 180 \] \hspace{1cm} Reject \( H_0 \) if \( t < -t_{\alpha} \)
In general p-values are given by:

Let $t_{\text{calc}} = t$ calculated

$H_a: \mu \neq 180$ \hspace{1cm} p-value = $2 \times P(t > |t_{\text{calc}}|)$

$H_a: \mu > 180$ \hspace{1cm} p-value = $P(t > t_{\text{calc}})$ \hspace{1cm} prob'y in direction of $H_a$

$H_a: \mu < 180$ \hspace{1cm} p-value = $P(t < t_{\text{calc}})$ \hspace{1cm} prob'y in direction of $H_a$