In problems where you need to choose form numbered choices, you can just circle your choice and then briefly explain why.

For some questions you may leave answers as unsimplified numerical expressions. The expressions could include mathematical notation such as √, \log_{10}, etc. For example

\[ y = \frac{1.2 + 1.3 + 1.8}{3} \quad \text{and} \quad s = \sqrt{\frac{(1.2 - \bar{y})^2 + (1.3 - \bar{y})^2 + (1.8 - \bar{y})^2}{2}} \]

would be fine unsimplified numerical expressions. The second expression is fine because \( \bar{y} \) had been given a numerical expression already. In these cases you do not need to evaluate or simplify the expression.
(1) (11 points) For their senior design project three Mechanical Engineering students investigated the potential savings in energy costs from installing solar absorbing blinds compared to having no blinds. Two small, 15’ x 15’ insulated sheds were built to the same specifications for this experiment, transported to Duluth, and set up in the same open field. They installed the solar absorbing blinds in one of the sheds and then recorded the energy consumption in both sheds.

(a) What is the biggest weakness of this experiment's design? (I'm not saying this is a bad experiment. Given cost and time constraints, the perfect experiment is never performed, but we want to recognize and handle potential weaknesses as well as possible.) For this and any word answers, you don't have to write extensively; just give clear, precise answers. You can give me more than one weakness if you want, but one particular weakness is what I am looking for.

(b) In light of the potential problems in making this comparison, what would you do to plan an experiment with only two sheds for making the comparison of energy consumption with and without the blinds as reliable as possible?
(2) (9 points)

(a) Under what conditions do we use the pooled or Satterthwaite's degrees of freedom, df, for the independent means t-test?

(b) The figure below is from this published paper.

*Figure 2.* Percentages of CD16-, CD69-, and VLA-1-positive peripheral blood monocytes of 10 healthy controls (contr) and 23 untreated patients with active pulmonary sarcoidosis (sarc). Values are presented as percentage of CD14^+ cells. Horizontal bars indicate mean values. Student t test (**p < 0.0001).

If we perform a t-test comparing the means for control (control) and sarc for CD16, should we use pooled degrees of freedom, df, for the test or Satterthwaite's approximation for df? Why?
(3) (16 points) The figure below is from this published paper. The survival percents (%) are the percents of surviving cells from that patient under two conditions.

The antiviral factor APOBEC3G enhances the recognition of HIV-infected primary T cells by natural killer cells
Jason M Norman, Michael Mashiba, Lucy A McNamara, Adewunmi Onafuwa-Nuga, Estelle Chiari-Fort, Wenwen Shen & Kathleen L Collins
Nature Immunology 12, 975–983 (2011) doi:10.1038/ni.2087

These data are paired measurements from 5 patients.

(a) How do you think the p-value using the paired t-test would compare to the p-value if we used the independent means two-sample t-test instead? Choose one of the following.

1. The p-value from the paired t-test would be quite a bit smaller than the p-value from the independent means t-test.
2. The p-value from the paired t-test would be about the same as the p-value from the independent means t-test.
3. The p-value from the paired t-test would be quite a bit larger than the p-value from the independent means t-test.

Why did you chose this answer?

(b) If we designed a new similar study with patients where we measure survival % of cells under two conditions, which do you think would have more power,

1. a paired design and paired t-test
   or
2. an independent means design and independent means t-test?

Why do think so?

(c) How many df would we use in the t-table for the paired t-test or confidence interval with these data?
The data below are from the article *Comparison of massage group and control group for both Spielberger STAI scores and changes in STAI score from baseline, at post treatment assessments.*


(a) The values given below are the sample mean, \( \bar{y} \), and standard deviation, S, for the massage group.

\[
\begin{array}{ccc}
\bar{y} & S & n \\
2.74 & 3.10 & 50 \\
\end{array}
\]

Write an unsimplified numerical expression for the estimated standard error of the mean for the massage group mean.

\[
SE_{\bar{y}} = \]

(b) Massage group                  Control group

\[
\begin{array}{ccc}
\bar{y} & S & n \\
2.74 & 3.10 & 50 \\
-0.28 & 3.43 & 43 \\
\end{array}
\]

Write unsimplified numerical expressions for the calculated t-value for testing \( H_0: \mu_{\text{Massage}} - \mu_{\text{Control}} = 0 \)

vs \( H_a: \mu_{\text{Massage}} - \mu_{\text{Massage}} \neq 0 \). This is the calculated t that we would compare to the t-table in deciding whether we reject \( H_0 \). Assume equal population variances.

(c) If we assume equal variances, what degrees of freedom, df, would be used for the t-test?
(d) Give an unsimplified numerical expression for a 1-sided 95% upper bound interval for \( \mu_{\text{Massage}} - \mu_{\text{Control}} \). You may refer back to any expressions you hand in part (b).

(5) (18 points)

(a) For this 2-tailed test, give a range for the p-value based on the t-table. Show briefly how you found this p-value. Based on this p-value, would we reject the null hypothesis at the \( \alpha = 0.05 \) level?

(b) What would the p-value be if the alternative was \( H_a: \mu_d > 0 \)?

(c) Which test would have more power if \( \mu_d = 3.0 \),
   1. the two-sided test \( H_0: \mu_d = 0 \) vs \( H_a: \mu_d \neq 0 \) or
   2. the one-sided test \( H_0: \mu_d \leq 0 \) vs \( H_a: \mu_d > 0 \)?

Briefly, how did you decide?
Researchers plan to compare cortisol levels for abstaining (A) smokers and non-abstaining (N) smokers. The null hypothesis is \( H_0: \mu_A - \mu_N = 0 \) and the alternative hypothesis is \( H_a: \mu_A - \mu_N \neq 0 \). The researchers want a 99% chance of finding a statistically significant difference between the groups (rejecting \( H_0 \)) if on average the difference between population means mean cortisol levels is \( |\Delta| = |\mu_A - \mu_N| = 5.0 \). They also want only a 5% chance of finding a statistically significant difference between the groups (rejecting \( H_0 \)) if there really is no difference between population mean cortisol levels, \( \mu_A - \mu_N = 0 \). Based on previous data, they will use a standard deviation of \( \sigma = 12 \). Their design will have \( n \) subjects in group A and \( n \) subjects in group B. They plan to assume equal variances.

(a) Give an unsimplified numerical expression for the required number of subjects required per treatment group, \( n \), using a Z test.

(b) How would the answer from the expression in part (a) compare to the required sample size if we found the required sample size for a t-test? Choose one answer.

1. The required sample size, \( n \), for the Z test will be less than the required sample size \( n \), for the t-test.
2. The required sample size, \( n \), for the Z test will be same as the required sample size \( n \), for the t-test.
3. The required sample size, \( n \), for the Z test will be greater than required sample size, \( n \), for the t-test.

Briefly, why did you choose this answer?