Stat 3411  Old Exams  Chapter 5

(1)  (a)  $0.1587$  
     (b)  $\frac{10!}{3!7!} (0.1587)^3 (1-0.1587)^7$

(2)  $1.96\sigma = 0.002$  
     $\sigma = \frac{0.002}{1.96}$

(3)  
     (a)  $P(X > 10) = 1 - P(X \leq 10) = 1 - F(10)$  
          $= 1 - \left(1 - e^{-\left(\frac{10}{8}\right)^{2.3}}\right)$
     (b)  $F(x) = 0.90$  
          $1 - e^{-\left(\frac{x}{8}\right)^{2.3}} = 0.90$  
          $e^{-\left(\frac{x}{8}\right)^{2.3}} = 0.10$  
          $-\left(\frac{x}{8}\right)^{2.3} = \ln(0.10)$  
          $x = 8\left(-\ln(0.10)\right)^{1/2.3}$

(4)  This is essentially the ballast water question again.
     (a)  $\lambda = \frac{\text{windshield}}{10,000 \text{ miles}} = 2$  
          $1 - P(0) = 1 - e^{-2} = 0.86$
     (b)  $1 - P(0) = 1 - e^{-x/10,000} = 0.95$  
          $x = 29,957 \text{ miles}$

Note:  Given the answer to (a), the answer to (b) has to be more than 20,000 miles.

(5)  

W is number of successes in 10 trials
W ~ binomial 10  \( p = 0.3 \)
\( P(W=3) = \frac{10!}{3!7!} (0.3)^3 (0.7)^7 \)
(6) 
\[ F(x) = P(X \leq x) = 0.05 \]
\[ 1 - e^{-\frac{80}{x}} = 0.05 \]
\[ 1 - e^{-\frac{80}{100}} = 0.05 \]
\[ e^{-\frac{80}{100}} = 0.95 \]
\[ \frac{x}{100} = \ln 0.95 \]
\[ x = 100 \left( \ln 0.95 \right)^{\frac{1}{3}} \]
\[ x = 27.16 \]

(7) 
The number of occurrences in a given space or time is Poisson as long as the events occur at a fixed rate. A uniformly mixed ballast tank would have dino-flagellates at a fixed rate \( \lambda \) 0.1 per m\(^2\) throughout the tank. In 20 m\(^2\),
\[ \lambda = \frac{20 \, \text{m}^2}{20} \frac{\text{dino}\text{flag} - \text{rates}}{\text{m}^2} = 2 \]
\[ P(X = 1) = 1 - P(0) = 1 - e^{-2} = 0.8647 \]

(8) 

(9) 
\[ X = \# \text{ resins} < 20.3 \text{ out of 4} \]
\[ X \sim \text{ binomial} \]
\[ P = P(X < 20.3) = P(Z < \frac{20.3 - 20}{\sigma}) \]
\[ = P(Z < 0.3) = 0.9332 \]
\[ P(X = 2) = \frac{4!}{2!} \left(0.9332\right)^2 \left(1 - 0.9332\right)^1 \]
(10) \[ P( X \geq 1500 ) = 1 - P(X < 1500) = 1 - \left( 1 - e^{-1550/2000} \right) = e^{-1550/2000} \]

(11) (a) Poisson

(b) \[ \lambda = \frac{4 \text{ seeds}}{100 \text{ ft}^2} \times 5 \times 10 \text{ ft}^2 = 2 \text{ seeds} \quad X = \text{number of seeds} \quad P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-2} \]

(c) Geometric

(12) \[ P(-0.67 < Z < 0.67) = 0.4972 \]

(13) Binomial

(13) Midpoint = 0.2765 \[ \frac{0.0275 - 0.02765}{\sigma} = 2.58 \quad \sigma = 0.000058 \]

(14)

\[ \mu = E(x) = \text{weighted average of } x \]
\[ = 0 \times 0.20 + 2 \times 0.15 + 4 \times 0.05 = 2.9 \]
\[ \sigma^2 = E((x-\mu)^2) = \text{weighted average of } (x-\mu)^2 \]
\[ = (0-2.9)^2 \times 0.20 + (2-2.9)^2 \times 0.15 + (4-2.9)^2 \times 0.05 \]

Note: As a check, \( \mu = \text{center of gravity} \)

(15)

(a) Poisson - The number of completely random events in time or space.

(b) \( X = \# \) of accidents in 4 weeks.
\[ E(x) = \lambda = 0.5 \text{ accidents week}^{-1} \times 4 \text{ weeks} = 2 \text{ accidents} \]
\[ P(X \geq 1) = 1 - P(0) = 1 - e^{-0.5} = 0.55 \]
The text is not clearly visible, but it appears to be discussing statistical concepts and calculations. The first part seems to be a binomial distribution problem, while the second part looks like it's involving a probability distribution function. Detailed mathematical expressions and calculations are present, but the specific content is not clear due to the quality of the image.
(19) \( Z = 1.125 \) \( Z \) about 1.12 or 1.13 \( p = 0.1314 \) or 0.1292

(20)  
\[
P(Y \geq 2) = 1 - \left[ P(0) + P(1) \right] = 1 - \left[ e^{-1.3} + e^{-1.3} \times 2.6 \right]
\]

\[
E(Y + Z) = \lambda = E(Y) + E(Z) = 1.3 + 1.3 = 2.6
\]

\[
P(Y + Z = 3) = \frac{e^{2.6} \times 2.6^3}{3!}
\]

(21)  
\[
P(198 < X < 202) = P\left(\frac{198 - 201}{1.25} < Z < \frac{202 - 201}{1.25}\right) =
\]

\[
P(-2.4 < X < 0.8) = 0.7881 - 0.0082 = 0.7799
\]

(22)  
\[
P(\text{single cable breaks}) = P(X < 400) = P\left(Z < \frac{400 - 450}{50}\right) = P(Z < -1) = 0.1587
\]

\[
P(\text{support fails}) = 1 - P(\text{all 5 cables hold}) = 1 - (1 - 0.1587)^5
\]
\[ \lambda = \frac{0.5 \text{ ships} \times t \text{ hours}}{\text{hour}} = 0.5t \]

\[ P(\text{at least 1}) = 1 - P(0) = 1 - e^{-0.5t} \]

\[ 1 - e^{-0.5t} = 0.95 \]

\[ t = -2 \ln(0.05) = 6 \text{ hours} \]