(1) Tell when to reject $H_0$: $\mu_1 - \mu_2 = 0$ using a t-test where $t = \frac{\overline{y}_1 - \overline{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Answers would be of the form
- Reject $H_0$ when $t < -1.746$
- or maybe
- Reject $H_0$ when $|t| > 1.746$ (t < -1.746 or t > 1.746)
- or maybe
- Reject $H_0$ when $t > 1.746$
  etc

(a) $H_A$: $\mu_1 - \mu_2 \neq 0$, $\alpha = 0.05$, $n_1=8$, $n_2=7$

(b) $H_A$: $\mu_1 - \mu_2 < 0$, $\alpha = 0.05$, $n_1=8$, $n_2=7$

(2) Give the p-value for testing $H_0$: $\mu_1 - \mu_2 = 0$ in the following situations. Give the p-value as near as you can from the t-table without interpolating.

Answers would be of the form
- $0.10 < p < 0.05$
- or maybe
- $p < 0.001$
- or maybe
- $p > 0.8$
- etc

After finding the p-value in each case, tell whether to reject or not reject $H_0$ at the $\alpha = 0.05$ level.

(a) $H_A$: $\mu_1 - \mu_2 \neq 0$, $n_1=8$, $n_2=7$, $t = -2.20$

(b) $H_A$: $\mu_1 - \mu_2 < 0$, $n_1=8$, $n_2=7$, $t = -2.20$

(c) $H_A$: $\mu_1 - \mu_2 > 0$, $n_1=8$, $n_2=7$, $t = -2.20$
(3) The modulus of elasticity for lumber boards was measured 1 minute before stacking the lumber and 4 weeks after stacking the lumber. (“Time-Dependent Bending Properties of Lumber” *Journal of Testing and Evaluation* 1996: 187-193 in Devore *Probability and Statistics for Engineering and the Sciences*) The data below are the values for 6 boards where each board is measured twice, once before stacking and once 4 weeks after stacking.

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>17800</td>
<td>15100</td>
</tr>
<tr>
<td>17300</td>
<td>14700</td>
</tr>
<tr>
<td>13600</td>
<td>11400</td>
</tr>
<tr>
<td>13000</td>
<td>10100</td>
</tr>
<tr>
<td>17200</td>
<td>14600</td>
</tr>
</tbody>
</table>

(a) Find a 95% 2-sided confidence interval for the average difference in elasticity between the boards before stacking and 4 weeks after stacking.

(b) Test the null hypothesis of no difference between 1 minute and 4 weeks elasticity versus the alternative that the elasticity is less after 4 weeks of stacking. Give the p-value as far as you can tell from the t-table. Is there a significant difference between 1 minute and 4 week stacked boards at the $\alpha=0.05$ level?

(c) What do we have to assume in order for our confidence interval and p-value to be valid?

(d) Use the data to check whether this assumption is valid in this case.

(4) One technology for pipeline rehabilitation uses a flexible liner thread through the existing pipeline. (“Effect of Welding on High-Density Polyethylene Liner” *Journal of Materials in Civil Engineering* 1996: 94-100 in Devore *Probability and Statistics for Engineering and the Sciences*) The following are data on tensile strength (psi) of liner specimens when a certain fusion process was used and when the process was not used.

<table>
<thead>
<tr>
<th>No fusion</th>
<th>2478</th>
<th>2700</th>
<th>2655</th>
<th>2822</th>
<th>2511</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3149</td>
<td>3257</td>
<td>3213</td>
<td>3220</td>
<td>2753</td>
</tr>
<tr>
<td>n=10</td>
<td>$\bar{x} = 2903$</td>
<td>$s = 277$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fused</td>
<td>3027</td>
<td>3356</td>
<td>3359</td>
<td>3297</td>
<td>3125</td>
</tr>
<tr>
<td></td>
<td>2910</td>
<td>2889</td>
<td>2902</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n=8</td>
<td>$\bar{x} = 3108$</td>
<td>$s = 206$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find a 2-sided 95% confidence interval for the difference in average tensile strength between the two methods. Assume both methods have the same variance in tensile strengths.

(b) Test the null hypothesis of not difference between the mean tensile strengths of the two methods (fused & not fused) against the alternative that the means are not the same.
Again assume equal variances. Give the p-value as far as you can tell from the t-table. Is there a significant difference between 1 minute and 4 week stacked boards at the \( \alpha = 0.05 \) level? How is your answer here related to your answer in part (a)?

(5) The alternating current breakdown voltage of an insulating fluid indicates its electric strength. An article on testing practices for breakdown voltage testing of insulating fluids tested 48 insulating fluids of a particular type yielding the following summary statistics.

\[
\begin{align*}
\text{n} & = 48 \\
\bar{x} & = 54.7 \text{ kV} \\
s & = 5.23
\end{align*}
\]

(a) Give a 95\% lower confidence bound for the average breakdown voltage of this type of insulating fluid.

(b) Based on this interval we would _________________ (reject or accept) the null hypothesis \( H_0: \mu = 50 \) against the alternative \( H_a: \mu \neq 50 \) (\( > \), \( < \), or \( \neq \)) at the \( \alpha = \) ______ level.

Fill in the blanks above and briefly explain your reasoning.

(6) Light bulbs of a particular brand are advertised to have an average lifetime of 750 hours. To test this claim we check 25 bulbs and obtain the following results.

\[
\bar{X} = 728 \quad S = 38
\]

(a) Test the null hypothesis \( H_0: \mu = 750 \) versus \( H_a: \mu < 750 \) at the \( \alpha = 0.10 \) level. Do we reject \( H_0: \mu = 750 \) in favor of \( H_a: \mu < 750 \)?

(b) Give a one-sided confidence interval that we could use for checking \( H_0: \mu = 750 \) versus \( H_a: \mu < 750 \). You can leave your answer as an unsimplified numerical expression.

(7) Data are collected on tensile strengths of wire of lengths 25 cm and 30 cm. Ten pieces of wire were tested, 4 pieces of 25 cm length and 4 pieces of 30 cm length, resulting in the following data.

\[
\begin{align*}
\text{Group 1} & \quad \text{Group 2} \\
25 \text{ cm:} & \quad 4.00, 4.65, 4.70, 4.50 \quad 30 \text{ cm:} \quad 4.10, 4.50, 3.80, 4.60
\end{align*}
\]

(a) Find a 95\% confidence interval for the difference between the mean strengths \( \mu_1 - \mu_2 \).

(b) Using the appropriate table, find the p-value for testing \( H_0: \mu_1 - \mu_2 = 0 \) versus \( H_a: \mu_1 - \mu_2 \neq 0 \). Assume both populations have the same population variances.
(c) Based on your answer to (b), would we reject \( H_0 \) in favor of \( H_a \) at the \( \alpha = 0.05 \) level? Explain your reasoning.

(d) In what way is your answer to (a) consistent or inconsistent with your answer to (c)?

(8) Based on past experience, the yield point of a particular type of steel-reinforcing bar is thought to be normal with standard deviation \( \sigma = 100 \). We want to estimate the mean yield. How many samples, \( n \), do we need to test in order to estimate the mean yield point with \( \pm 20 \) precision and 99% confidence? That is, we want a 99% chance that our sample mean deviates from the true mean, \( \mu \), by no more than \( \pm 20 \).

(9) Tell when to reject \( H_0: \mu_1 - \mu_2 = 0 \) assuming equal variances.

Answers would be of the form
- Reject \( H_0 \) when \( t < -2.511 \) or maybe
- Reject \( H_0 \) when \( |t| > 1.746 \) (\( t < -1.746 \) or \( t > 1.746 \)) or maybe
- Reject \( H_0 \) when \( t > 2.075 \) or maybe …………

(a) \( H_A: \mu_1 - \mu_2 > 0, \alpha = 0.01, n_1=8, n_2=12 \) Explain briefly how you found your answer.

(b) \( H_A: \mu_1 - \mu_2 \neq 0, \alpha = 0.01, n_1=8, n_2=12 \) Explain briefly how you found your answer.

(10) Give the p-value for testing \( H_A: \mu_1 - \mu_2 = 0 \) in the following situations assuming the populations have equal variances. Give the p-value as near as you can from the t-table without interpolating.

Answers would be of the form
- \( 0.10 < p < 0.05 \) or maybe
- \( p < 0.001 \) or maybe
- \( p > 0.8 \) or maybe ………

After finding the p-value in each case, tell whether to reject or not reject \( H_0 \) at the \( \alpha = 0.05 \) level.

(a) \( H_A: \mu_1 - \mu_2 > 0, n_1=8, n_2=12, t = -1.89 \) Explain briefly where you found your answer.

(b) \( H_A: \mu_1 - \mu_2 \neq 0, n_1=8, n_2=12, t = -1.89 \) Explain briefly where you found your answer.

(11) To compare measurements of deuterium using an isotopic method and a test-weighing method, deuterium levels in 6 samples were obtained. For each sample the deuterium level was determined once with the isotopic methods and once with the test-weighing method.

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotopic</td>
<td>1418</td>
<td>1561</td>
<td>2169</td>
<td>1760</td>
<td>1479</td>
<td>1281</td>
<td>1954</td>
</tr>
<tr>
<td>Test-weighing</td>
<td>1254</td>
<td>1336</td>
<td>2000</td>
<td>1318</td>
<td>1342</td>
<td>1124</td>
<td>1604</td>
</tr>
</tbody>
</table>
Explain how you would test the null hypothesis that on average there is no difference between the methods versus the alternative that on average there is a difference in the methods. You do not need to do any calculations. Just explain the steps you would take in testing this hypothesis at the $\alpha = 0.05$ level.

(a) How would you compute the t value that you would compare to the t table? Include the formula you would use.

(b) To what tabled value would you compare your computed t to decide whether to reject the null hypothesis? (The answer here would be similar to problem 4 above.)

12. Give the p-value for testing $H_0: \mu = 500$ with a z-score for the following situations. Explain briefly how you determined this answer.

(a) $H_A: \mu \neq 500$, $z = -1.55$

(b) $H_A: \mu < 500$, $z = -1.55$

(c) $H_A: \mu > 500$, $z = -1.55$

13. A company makes bearings that are supposed to be 1 cm in diameter. If a 95% confidence interval for the mean diameter is 0.97 cm to 1.02 cm, the p-value for testing $H_0: \mu = 1$ versus $H_A: \mu \neq 1$ will be ________________. Fill in the blank with either big or small. Briefly explain your answer.

14. The EPA has recently developed new criteria for sulfate levels in discharge water. The current criterion is a limit of 500 mg/L for undiluted discharge water. Suppose measurements of sulfate in the discharge water of a coal mining site can be viewed as a random sample from a normal population with standard deviation $\sigma = 20$ and mean $\mu = 490$ mg/L.

(a) What is the probability that the mean of a sample of size 4 will be above 500 mg/L?

(b) How large of a sample needs to be taken to be 95% certain that the sample mean is within ±2 mg/L of the population mean $\mu$? You can leave your answer as an unsimplified numerical expression.

15. The EPA criterion limit for copper in undiluted effluent is 5 $\mu$g/L. In monitoring the effluent, a test is performed of $H_0: \mu = 5$ versus $H_A: \mu > 5$. A p-value of 0.002 is obtained. A company manager asks for you to explain to them what a p-value of 0.002 means. Briefly, how would you explain a p-value such as this in words the manager would understand?

16. A company sells resistors than are nominally 100 ohm resistors. A sample of 25 resistors gives
\( \bar{X} = 101 \quad \quad S = 2 \)

(a) Find a 99% confidence interval for the population mean resistance of these resistors.

(b) Based on this confidence interval, we would _________ (reject or not reject) 
\( H_0 \) at the \( \alpha = _________ \) level when testing \( H_0: \mu = 100 \) versus \( H_A: \mu \neq 100 \).

(17) An article in the *Journal of Engineering Technology* compared performances of pneumatic robot grippers. One aspect compared grip pressures for rectangular and circular bars. They measured 4 values for the rectangular bar and 5 values for the circular bar, resulting in the following results.

<table>
<thead>
<tr>
<th></th>
<th>Rectangular</th>
<th>Circular</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{X} )</td>
<td>82.6</td>
<td>87.4</td>
</tr>
<tr>
<td>( S )</td>
<td>5.3</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Find a 95% confidence interval for \( \mu_1 - \mu_2 \) assuming \( \sigma_1 = \sigma_2 \).

You can leave steps in your answer and your final answer as unsimplified numerical expressions.

(18) Measurements of strengths for 8 pieces of wire resulted in a sample mean of 4.30 kg and a sample standard deviation of 0.33 kg. Give an unsimplified numerical expression for the 95% lower confidence bound for the mean strength for this wire.

(19) Measurements of 821 particles from a Zircaloy material resulted in measured diameters with a sample mean of \( \bar{X} = 0.055 \) \( \mu \text{m} \) and a standard deviation of \( s = 0.028 \) \( \mu \text{m} \). Suppose we are testing \( H_0: \mu = 0.057 \) versus \( H_A: \mu \neq 0.057 \). Perform the test, and find the p-value to within a range from the tabled values. Specify what tabled values you compared to. What do you conclude about this null hypothesis?
Compression strengths of taconite pellets run around 560 pounds.

National Steel Pellet Company’s iron ore pellets have the following characteristics (FOB Mine):

- **Total Iron**: 65.85%
- **Silica (SiO2)**: 4.5%
- **Lime (CaO)**: 0.08%
- **Phosphorus**: 0.010

- **Size**: %+1/4", 96.5% (after tumble)
- **Compression Strength**: 560 pounds

Suppose we want to estimate the average compression strength of taconite pellets from a particular process. We anticipate that the standard deviation for compression strengths of these pellets will be about 30 pounds. Give an unsimplified numerical expression for how many pellets do we need to test in order to estimate the mean compression strength to within ±2 pounds with 95% certainty. That is, give an unsimplified numerical expression for the sample size would be needed to be 95% certain that $X$ falls within 2 pounds of $\mu$.

Someone at your company is interested in comparing resistivities of a type of wire at $0^\circ$ centigrade and $21.8^\circ$ centigrade. They have 7 measurements at $0^\circ$ and 7 measurements at $21.8^\circ$. Terry asked you to help find a 95% confidence interval for the average difference between resistivity at $0^\circ$ and $21.8^\circ$. What question or questions do you need to ask Terry before you can find the desired confidence interval? I'm looking for one particular question that you definitely need to ask.

The distances traveled for a sample of 10 drives using a titanium driver and a two-piece golf ball resulted in a sample mean of 205 yards and a sample standard deviation of 5 yards. You are planning another experiment with a different driver. If the standard deviation of drive distances is anticipated to be about 1.5 yards, how many drives are required to be 99% certain that the sample average distance is within 2 yards of the true, population average distance?