(1) \( a \)
\[ df = (8-1) + (6-1) = 13 \]
\[ H_1 > 2.16 \]
\[ b \]
\[ t < -1.771 \]

(2) \( a \)
\[ P(|t| > 2.00) \]
\[ 2 \times 0.01 < P < 2 \times 0.025 \]
\[ 0.02 \leq P \leq 0.05 \]
\[ -2.2 \]
\[ b \]
\[ P(t < -2.00) \]
\[ 0.01 \leq P \leq 0.025 \]
\[ c \]
\[ P(t > -2.00) \]
\[ 0.975 \leq P \leq 0.99 \]
(3) These are paired data.

The data are highly correlated so the paired analysis is much different than the incorrect unpaired.

<table>
<thead>
<tr>
<th>Board</th>
<th>Before</th>
<th>After</th>
<th>d = Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17800</td>
<td>15100</td>
<td>2700</td>
</tr>
<tr>
<td>2</td>
<td>17300</td>
<td>14700</td>
<td>2600</td>
</tr>
<tr>
<td>3</td>
<td>13600</td>
<td>11400</td>
<td>2200</td>
</tr>
<tr>
<td>4</td>
<td>13000</td>
<td>10100</td>
<td>2900</td>
</tr>
<tr>
<td>5</td>
<td>17200</td>
<td>14600</td>
<td>2600</td>
</tr>
</tbody>
</table>

The boards are obviously less elastic after stacking. The confidence interval will obviously not include 0, the t-value will be really big, and the p-value will be small.
(a) \( df = 4 \quad \bar{d} = 2600 \quad S_d = 255 \)

\[
2600 \pm 2.776 \left( \frac{255}{\sqrt{5}} \right)
\]

(b) \[
T = \frac{\bar{d} - \mu}{S_d/\sqrt{n}} = \frac{2600}{255/\sqrt{5}} = 22.8
\]

\( p < 0.0005 \) from table.

\( p \approx 0 \) for \( T \) this big.

There is huge evidence that the boards lose elasticity when stacked.

Incorrectly doing this as unpaired:

Conf Interval \( 2600 \pm 3324 \)

vs. Paired \( 2600 \pm 317 \)

\[
T = 1.80 \quad \text{unpaired}
\]
\[
T = 22.8 \quad \text{paired}
\]
(c) The differences are normal and independent. If you did an unpaired test, the answer is normal populations, independent samples and equal variances. For unpaired tests check normal plots for both groups on same graph.

\[ S_p^2 = \frac{(15-1) \cdot 279^2 + (8-1) \cdot 206^2}{(10-1) + (8-1)} = \frac{9 + 7}{16} \text{ df} \]

\[ S_p = \sqrt{S_p^2} = 248.44 \]

\[ 3108 - 2903 \pm 2.120 \times 248.44 \sqrt{\frac{1}{10} + \frac{1}{8}} \]

\[ -44.8 \text{ to } 454.8 \]

(b) \[ t = \frac{\bar{X}_F - \bar{X}_{NF}}{S_p \sqrt{\frac{1}{10} + \frac{1}{8}}} = 1.074 \]

\[ 2 \times 0.05 < P < 2 \times 0.1 \]

\[ 0.1 < P < 0.2 \]

This is not convincing evidence of a difference. The data are not real.
unsaid unlikely if \( \mu = \mu_0 \). Since 0 is in the confidence interval, the p-value is above 0.05. We could not conclude that the fused liners have the same strength as unfused liners. No difference, or is one possibility based on the confidence interval but a difference as large as 45% is 15% stronger, is also an unrejected possibility.

(5) (a) 54.7 \(- 1.684 \times 5.23 / \sqrt{48} \)

(b)

We have good evidence that \( \mu \) is above 50, since 50 is not in the interval. We reject \( H_0: \mu = 50 \) in favor of the alternative \( H_a: \mu > 50 \) at the \( \alpha = 0.05 \) level.

(6) (a) \( t = \frac{728 - 750}{38 / \sqrt{25}} = -2.89 \)

Either (i) compare the computed \( t \) to the \( t \) tabled value or (ii) compare the p-value to 0.10.

(i) Reject \( H_0 \) if \( t < -1.318 \). -2.89 < -1.318 => Reject \( H_0 \) at the \( \alpha = 0.10 \) level.

(ii) 0.001 < \( p < 0.005 \). \( p < 0.10 \) => Reject \( H_0 \) at the \( \alpha = 0.10 \) level.

(b) \(-\infty \) to \( 728 + 1.318 \times 38 / \sqrt{25} \) \(-\infty \) to 738. It does appear that \( \mu < 750 \).

(7) (a) You could either assume equal variances or not. I took either approach, since I didn’t tell you what to do and the standard deviations are pretty similar. The df comes out to 6 or about
6 either way for these data. When \( n_1 = n_2 \) the standard error, \( SE_{\bar{x}_1 - \bar{x}_2} \), comes out to the same number either way, in this case 0.244.

\[
4.4625 - 4.25 \pm 2.447 \times 0.244 \quad (-0.39, 0.81)
\]

(b) \( t = \frac{4.4625 - 4.25}{\sqrt{0.1195 \left( \frac{1}{4} + \frac{1}{4} \right)}} = 0.87 \quad p > 0.20 \quad (c) \quad p > 0.05. \) Do not reject \( H_0. \)

(d) \( \mu_1 - \mu_2 = 0 \) is in the 95% confidence interval. This is consistent with not rejecting \( H_0 \) at the \( \alpha = 0.05 \) level.

(8) Solve \( 2.576 \frac{100}{\sqrt{n}} \leq 20 \quad n = 166 \)

(9) (a) Reject \( H_0 \) if \( t > 2.552 \) \quad (b) Reject \( H_0 \) if \(|t| > 2.878 \)

(10) (a) \( 0.95 < p < 0.975 \) \quad Do not reject \( H_0. \)

(b) \( 0.05 < p < 0.10 \) \quad Do not reject \( H_0. \)

(11) (a) Use a paired t-test. Find differences, \( d \), for each pair. \( t = \frac{\bar{d} - 0}{S_d / \sqrt{n}} \).

(b) Reject \( H_0 \) if \(|t| > 2.447 \)

(12)
In part c) with $z = -1.55$ (corresponding to a mean less than 500) we have no reason whatsoever to favor $H_a: \mu > 500$, so the p-value is big.
1) Since \( \mu = 1 \) is well within the confidence interval, we have little evidence to reject \( H_0: \mu = 1 \) so the p-value is big.

(3) \( z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{500 - 490}{20/\sqrt{14}} = \frac{10}{2.0} = 5 \)

(6) \( \Delta = (\frac{\bar{x} - \mu}{\Delta})^2 = (\frac{1.96 \times 20}{2})^2 = 384.2 \)

Round up to 385

2) I gave some points for correctly describing the meaning of a p-value and some points for correctly interpreting what this p-value tells us about the discharge.
There is only a 2/1000 chance of having our effluent test come out this bad if the true mean copper concentration in the effluent water is really 5 μg/L. This is strong evidence that our copper concentration is actually above 5 μg/L.
Further comments: The p-value is the probability of seeing data like this (or worse) if $H_0$ is true. The p-value is not the probability that $H_0$ is true. $H_0$ is either true or not; it is not a random quantity with some probability, at least in the classical, frequentist statistical perspective. If you want to say something like $P(\mu > 500) = 0.982$, you need to learn about “Bayesian” statistics. My opinion is that beyond the semantics of saying $P(\mu > 500) \approx 0.982$ or whatever Bayesian methods have some definite advantages, but this is hotly debated.

The p-value is related to $P(\text{data} \mid H_0 \text{ true})$ Bayesian statistical methods attempt to find $P(\text{H}_0 \text{ true} \mid \text{data})$. More exactly the p-value is

$$P(\text{data this bad or worse for } H_0 \mid H_0 \text{ true})$$

(16)

$$101 \pm 2.797 \frac{2}{\sqrt{25}} \quad df = 24$$

Do not reject (accept) $H_0$ at the $\alpha = 0.01$ level.
The traditional cutoff for having $p$ small enough to reject the null hypothesis is $p < 0.05$. Since $p < 0.05$ here, we reject the null hypothesis at the $\alpha = 0.05$ level.

95% certain that $\bar{X}$ deviates from $\mu$ by no more than $1.96$ SE's or $1.96\left(\frac{30}{\sqrt{n}}\right)$.

$$1.96\left(\frac{30}{\sqrt{n}}\right) = 2 \text{ pounds}$$

$$n = \left(\frac{1.96*30}{2}\right)^2 \approx 864$$

You need to know if the data are paired or not. Was the same piece of wire measured twice, once at each temperature, or do we have 14 independent measurements.

Solve $2.576\sqrt{\frac{1.5^2}{n}} = 2$