Example: Suppose, for a 3 minute egg timer, \( n = 10 \) and \( \alpha = 0.05 \)

\[ H_0: \mu = 180 \text{ seconds} \quad H_a: \mu \neq 180 \]

Reject \( H_0 \) if \( |T| = \left| \frac{\bar{y} - 180}{s/\sqrt{n}} \right| > 2.26 \)

Say \( \bar{y} = 182.1 \quad s = 3.2 \quad n = 10 \quad SE_{\bar{y}} = \frac{3.2}{\sqrt{10}} = 1.01 \quad T = \frac{182.1 - 180}{1.01} = 2.08 \)

Do not reject \( H_0: \mu = 180 \) for \( \alpha = 0.05 \) since \( T < 2.26 \)

Reject \( H_0: \mu \neq 180 \) for \( \alpha = 0.10 \) since \( T > 1.83 \)

\[ 2 \times 0.025 < \text{p-value} < 2 \times 0.050 \]
\[ 0.05 < \text{p-value} < 0.10 \]

Reject \( H_0 \) if \( \text{p-value} < \alpha \). If \( \alpha = 0.05 \), don’t reject \( H_0 \). If \( \alpha = 0.10 \), do reject \( H_0 \).
If we have $H_a: \mu > 180$ \[ T = \frac{\bar{y} - 180}{SE} \] Reject $H_0$ if $t > 1.833$

If $T = 2.08$, reject $H_0$ if $\alpha = 0.05$ but not if $\alpha = 0.025$ ($t_{0.025} = 2.262$)

P-value: $0.025 < p - value < 0.05$

If $T$ is on the reject side of $H_0$:
1-sided p-value $= \frac{1}{2} \times$ 2-sided p-value

If $T$ is so far from reject that it’s on the other side of $H_0$:
1-sided p-value $= 1 - \frac{1}{2} \times$ 2-sided p-value

$H_a: \mu < 180$

1-sided p-value $= 1 - \frac{1}{2} \times$ (2-sided p-value)

0.95 $< p$-value $< 0.975$
In general rejection regions are given by:

\[ H_0 : \mu = 180 \quad \text{Reject } H_0 \text{ if } |t| > t_{\alpha/2} \]

\[ H_a : \mu \neq 180 \]

\[ H_a : \mu > 180 \quad \text{Reject } H_0 \text{ if } t > t_\alpha \]

\[ H_a : \mu < 180 \quad \text{Reject } H_0 \text{ if } t < -t_\alpha \]
In general p-values are given by:

Let \( T_{\text{calc}} = T \) calculated

\[
\begin{align*}
H_a: \mu &\neq 180 \quad \text{p-value} = 2 \times P(T > |T_{\text{calc}}|) \quad \text{prob’y in direction of } H_a \\
H_a: \mu &> 180 \quad \text{p-value} = P(T > T_{\text{calc}}) \\
H_a: \mu &< 180 \quad \text{p-value} = P(T < T_{\text{calc}})
\end{align*}
\]

\( \text{p-value} = \text{Prob’y in direction of } H_a \text{ regardless of which side of zero } T_{\text{calc}} \text{ is on.} \)
Confidence Intervals

\[ \bar{x} \pm t_{a/2} SE = \bar{x} = 182.1 \quad n = 10 \quad s = 3.2 \quad SE = \frac{s}{\sqrt{n}} = 1.01 \]

182.1 ± 2.262(1.01)
182.1 ± 2.28
179.8 to 184.4

Important point: Finding 180 in 95% confidence interval is equivalent to not rejecting \( H_0 : \mu = 180 \) versus \( H_a : \mu \neq 180 \) at the \( \alpha = 5\% \) level.

The confidence interval is much more useful than the p-value.

We knew before taking any data that \( \mu \) isn’t exactly 180.

We don’t really care if \( \mu = 180 \).

We want to know potentially how far the mean is from 180.

Even more likely, we want to know the probability that the timer results in a time "close enough" to 180.

"Close enough" depends on the application.

"An approximate answer to the exact question is always better than the exact answer to an approximate question." John Tukey.

An answer to the wrong question is even worse.