Tests and Confidence Intervals: Basic Facts

- \( SE_{\text{Estimate}} = \sqrt{\text{Var}(\text{Estimate})} \) with estimated variances from sample statistics

- \( \text{Var}(\bar{X}) = \frac{\sigma^2}{n} \)

- For independent variables \( \text{Var}(U \pm V) = \text{Var}(U) + \text{Var}(V) \)
  \[
  \text{Var}(\bar{X}_1 - \bar{X}_2) = \text{Var}(\bar{X}_1) + \text{Var}(\bar{X}_2) \\
  \text{Var}(\hat{p}_1 - \hat{p}_2) = \text{Var}(\hat{p}_1) + \text{Var}(\hat{p}_2)
  \]

- 95% confidence intervals are
  \[
  \text{Estimate} \pm t \times SE_{\text{Estimate}} \quad \text{Estimate} - t \times z_{0.025} \times SE_{\text{Estimate}} \quad \text{Estimate} + t \times z_{0.025} \times SE_{\text{Estimate}}
  \]
  For means use \( t \). For proportions, use \( z \).

- To test \( H_0: \text{Parameter} = \#_0 \), \( z_{\text{calculated}} \) or \( t_{\text{calculated}} = \frac{\text{Estimate} - \#_0}{\text{SE}_{\text{Estimate}}} = \text{SE's from } H_0 \)

- For rejecting \( H_0: \text{Parameter} = \#_0 \) for \( \alpha=0.05 \), the following are equivalent.
  - Reject \( H_0: \text{Parameter} = \#_0 \) in favor of
    - \( H_a: \text{Parameter} \neq \#_0 \) if \( |t \text{ or } z_{\text{calculated}}| > t \text{ or } z_{0.025} \)
    - \( H_a: \text{Parameter} > \#_0 \) if \( t \text{ or } z_{\text{calculated}} > t \text{ or } z_{0.05} \)
    - \( H_a: \text{Parameter} < \#_0 \) if \( t \text{ or } z_{\text{calculated}} < -t \text{ or } z_{0.05} \)
  - Or similarly for \( z_{\text{calculated}} \)

- Reject \( H_0 \) if p-value < \( \alpha = 0.05 \)

- Reject \( H_0 \) if the associated confidence interval (CI) does not include \( \# \)
  - Reject \( H_0 \) in favor of \( H_a: \text{Parameter} \neq \#_0 \)
    if 2-sided 95% CI doesn't include \( \#_0 \)
  - Reject \( H_0 \) in favor of \( H_a: \text{Parameter} > \#_0 \)
    if 1-sided 95% interval [Lower Bound, \( \infty \)] is completely above \( \#_0 \)
  - Reject \( H_0 \) in favor of \( H_a: \text{Parameter} < \#_0 \)
    if 1-sided 95% interval [\( -\infty \), Upper Bound] is completely below \( \#_0 \)

- The p-value for testing \( H_0: \text{Parameter} = \#_0 \) is
  - \( 2P(T > |t \text{ or } z_{\text{calculated}}|) \) for \( H_a: \text{Parameter} \neq \#_0 \)
  - \( P(T > t \text{ or } z_{\text{calculated}}) \) for \( H_a: \text{Parameter} > \#_0 \)
  - \( P(T < t \text{ or } z_{\text{calculated}}) \) for \( H_a: \text{Parameter} < \#_0 \)
**For Means**
- Use sample standard deviations to estimate $\sigma$'s in standard errors.
- For two independent groups with $\sigma_1^2 = \sigma_2^2 = \sigma^2$

  The pooled estimate of $\sigma^2$ is
  
  $S_p^2 = \text{Estimate of } \sigma^2 \text{ in } SE_{x_1-x_2}$
  
  $S_p^2 = \text{weighted average of } S_1^2 \text{ and } S_2^2 \text{ weighted by degrees of freedom}$
  
  $df$ for $S_1^2 = n_1 - 1$. $df$ for $S_p^2 = df_1 + df_2$

- For two independent groups without assuming equal variances

  Use $S_1^2$ and $S_2^2$ separately for $\sigma_1^2$ and $\sigma_2^2$ in $SE_{x_1-x_2} = \sqrt{SE_1^2 + SE_2^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

  $df = \text{Satterthwaite formula}$

**For proportions**
- $\hat{p} = \overline{X}$ where $X_i$ are 0's and 1's
  
  $\sigma^2 = p(1-p)$
  
  $X_i$'s are not normal.

  Use $z$ (not $t$) as long as
  
  $np = \# \text{ of times event happened and } n(1-\hat{p}) = \# \text{ of times event didn't happened \ both } \geq 5$.

- $Var(\hat{p}) = \frac{\sigma^2}{n} = \frac{p(1-p)}{n}$

- Use $\hat{p}$ for $p$ in SE's

- When testing $H_0$ use SE's calculated for case where $H_0$ is true.

  $H_0: p = \#$. Use $\#$ for $p$

  $H_0: \hat{p}_1 = \hat{p}_2 = p$. Use combined estimate of $p$ over both groups.