For the regression of $Y = \ln \text{Life}$ versus $X = \ln \text{Speed}$

ANOVA

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>32.26</td>
<td>32.26</td>
<td>497.01</td>
<td>1.467E-14</td>
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<tr>
<td>Residual</td>
<td>18</td>
<td>1.168</td>
<td>0.06491</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>33.428</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find $R^2$. Confirm that this is consistent with the degree of fit in the plotted data. 0.965
(b) Find the estimated value of \( \sigma^2 \), the variance of y values for a fixed x value. 0.06491

(c) Find the estimated value of \( \sigma \), the standard deviation of y values for a fixed x value. Confirm from the plotted values that this value looks about right. 0.2548

For the regression of \( Y = \text{Ln Life} \) versus \( X = \text{Ln Speed} \)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Intercept</th>
<th>Ln Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>34.344</td>
<td>-5.186</td>
</tr>
</tbody>
</table>

(d) Find estimated Ln Life when Speed = 400. Confirm from the plot that the value looks about right. 3.274

(e) Find estimated Life when Speed = 400. Confirm from the plot that the value looks about right. 26.425

Equations with Final Exam

\[
V_{\text{ar}}(b_1) = \frac{\sigma^2}{(n-1)*S^2_x} \\
V_{\text{ar}}(\hat{\gamma}) = V_{\text{ar}}(\bar{y} + (x-\bar{x})*b_1) = V_{\text{ar}}(\bar{y}) + V_{\text{ar}}((x-\bar{x})*b_1) = \frac{\sigma^2}{n} + (x-\bar{x})^2 V_{\text{ar}}(b_1) \\
V_{\text{ar}}(y_{new} - \hat{\gamma}) = V_{\text{ar}}(y_{new} - \hat{\gamma}) = V_{\text{ar}}(y_{new}) + V_{\text{ar}}(\hat{\gamma}) = \sigma^2 + V_{\text{ar}}(\hat{\gamma})
\]

(f) Find the estimated variance of the slope, \( b_1 \). 0.05411

(g) Find the estimated standard error of the slope, \( b_1 \). 0.2326

(h) Find the 95% confidence interval for the slope, \( b_1 \).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>34.344</td>
<td>1.482</td>
<td>23.17</td>
<td>7.493E-15</td>
<td>31.230</td>
</tr>
<tr>
<td>Ln Speed</td>
<td>-5.186</td>
<td>0.2326</td>
<td>-22.29</td>
<td>1.467E-14</td>
<td>-5.674</td>
</tr>
</tbody>
</table>

(i) For \( y = \text{Ln Life} \), find the estimated variance of \( \hat{\gamma} \) when speed = 400. 0.01091

(j) For \( y = \text{Ln Life} \), find the estimated standard error of \( \hat{\gamma} \) when speed = 400. 0.104

(k) Find the 95% confidence interval for the mean of Ln Life when speed = 400. Confirm from the plot that the value looks about right.

\[
3.055 \quad 3.494
\]

(l) Find the 95% confidence interval for the mean Life when speed = 400.

\[
21.22 \quad 32.91
\]

(m) For \( y = \text{Ln Life} \) and speed=400, find the estimated \( V_{\text{ar}}(y_{new} - \hat{\gamma}) \). 0.07581
(n) For $y = \ln \text{Life}$ and speed = 400, find the estimated standard error of $y_{new} - \hat{y}$. 0.2753

(o) Find the 95% prediction interval for the next measured value of Ln Life when speed = 400. Confirm from the plot that the value looks about right.

2.696    3.853

(p) Find the 95% confidence interval for the next measured value of Life when speed = 400.

14.82    47.12

(q) What is the average change in Ln Life if speed is increased by 25%? -1.157

(r) What is the ratio of predicted tool lives at an initial speed and at a 25% higher speed? $\frac{\text{Life at speed}\ 1.25 \cdot S_1}{\text{Life at speed}\ S_1}$

0.3144

(s) What is the % change in predicted tool life if speed is increased by 25%? -68.6%

(t) Write the fitted equation relating tool life to speed. Write the equation in simplest terms.

(u) Use your equation in (t) to find estimated lives for speed = 400 and for a 25% increase in speed. Confirm your answers from the recorded data given above.

(v) Use the values in part (u) to confirm your ratio in part (r).

<table>
<thead>
<tr>
<th>Speed</th>
<th>Life</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>26.425</td>
<td>0.3144</td>
</tr>
<tr>
<td>500</td>
<td>8.308</td>
<td></td>
</tr>
</tbody>
</table>

(w) An approximate percent change of life when speed increases by 25 % would be $-5.186 \times 25\% = -130\%$.
Why isn't this a good approximation for the effect on tool life with a 25% increase in speed?