For the regression of $Y = \ln \text{Life}$ versus $X = \ln \text{Speed}$

<table>
<thead>
<tr>
<th>Speed</th>
<th>Life</th>
<th>Ln Speed</th>
<th>Ln Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1.00</td>
<td>6.685</td>
<td>0.000</td>
</tr>
<tr>
<td>800</td>
<td>0.90</td>
<td>6.685</td>
<td>-0.105</td>
</tr>
<tr>
<td>800</td>
<td>0.74</td>
<td>6.685</td>
<td>-0.301</td>
</tr>
<tr>
<td>800</td>
<td>0.66</td>
<td>6.685</td>
<td>-0.416</td>
</tr>
<tr>
<td>700</td>
<td>1.00</td>
<td>6.551</td>
<td>0.000</td>
</tr>
<tr>
<td>700</td>
<td>1.20</td>
<td>6.551</td>
<td>0.182</td>
</tr>
<tr>
<td>700</td>
<td>1.50</td>
<td>6.551</td>
<td>0.405</td>
</tr>
<tr>
<td>700</td>
<td>1.60</td>
<td>6.551</td>
<td>0.470</td>
</tr>
<tr>
<td>600</td>
<td>2.35</td>
<td>6.397</td>
<td>0.854</td>
</tr>
<tr>
<td>600</td>
<td>2.65</td>
<td>6.397</td>
<td>0.975</td>
</tr>
<tr>
<td>600</td>
<td>3.00</td>
<td>6.397</td>
<td>1.099</td>
</tr>
<tr>
<td>600</td>
<td>3.60</td>
<td>6.397</td>
<td>1.281</td>
</tr>
<tr>
<td>500</td>
<td>6.40</td>
<td>6.215</td>
<td>1.856</td>
</tr>
<tr>
<td>500</td>
<td>7.80</td>
<td>6.215</td>
<td>2.054</td>
</tr>
<tr>
<td>500</td>
<td>9.80</td>
<td>6.215</td>
<td>2.282</td>
</tr>
<tr>
<td>500</td>
<td>16.50</td>
<td>6.215</td>
<td>2.803</td>
</tr>
<tr>
<td>400</td>
<td>21.50</td>
<td>5.991</td>
<td>3.068</td>
</tr>
<tr>
<td>400</td>
<td>24.50</td>
<td>5.991</td>
<td>3.199</td>
</tr>
<tr>
<td>400</td>
<td>26.00</td>
<td>5.991</td>
<td>3.258</td>
</tr>
<tr>
<td>400</td>
<td>33.00</td>
<td>5.991</td>
<td>3.497</td>
</tr>
</tbody>
</table>

Mean 6.3677 1.3231
St Dev 0.2513 1.3264

For the regression of $Y = \ln \text{Life}$ versus $X = \ln \text{Speed}$

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>32.26</td>
<td>32.26</td>
<td>497.01</td>
<td>1.467E-14</td>
</tr>
<tr>
<td>Residual</td>
<td>18</td>
<td>1.168</td>
<td>0.06491</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>33.428</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find $R^2$. Confirm that this is consistent with the degree of fit in the plotted data. 0.965
(b) Find the estimated value of $\sigma^2$, the variance of y values for a fixed x value. $0.06491$

(c) Find the estimated value of $\sigma$, the standard deviation of y values for a fixed x value. Confirm from the plotted values that this value looks about right. $0.2548$

For the regression of $Y = \ln \text{Life}$ versus $X = \ln \text{Speed}$

\[
\begin{array}{l}
\text{Coefficients} \\
\hline
\text{Intercept} & 34.344 \\
\text{Ln Speed} & -5.186 \\
\hline
\end{array}
\]

(d) Find estimated $\ln \text{Life}$ when $\text{Speed} = 400$. Confirm from the plot that the value looks about right. $3.272$

\[
34.344 - 5.186 \times \ln(400)
\]

(e) Find estimated Life when $\text{Speed} = 400$. Confirm from the plot that the value looks about right. $26.4$

\[
e^{3.272}
\]

Equations with Final Exam

\[
\begin{align*}
Var(b_1) &= \frac{\sigma^2}{(n-1) S_x^2} \\
Var(\hat{y}) &= Var(\bar{y} + (x - \bar{x}) b_1) = Var(\bar{y}) + Var((x - \bar{x}) b_1) = \frac{\sigma^2}{n} + (x - \bar{x})^2 Var(b_1) \\
Var(y_{new} - \hat{y}) &= Var(y_{new} - \hat{y}) = Var(y_{new}) + Var(\hat{y}) = \sigma^2 + Var(\hat{y})
\end{align*}
\]

(f) Find the estimated variance of the slope, $b_1$. $0.0541$

\[
Var(b_1) = \frac{\sigma^2}{(n-1) S_x^2} \approx \frac{MS \text{ Residual}}{(n-1) S_x^2} = \frac{0.06491}{(20-1) \times 0.2513^2}
\]

(g) Find the estimated standard error of the slope, $b_1$. $0.2326$

\[
SE_{b_1} = \sqrt{Var(b_1)} = \sqrt{0.0541}
\]

(h) Find the 95% confidence interval for the slope, $b_1$.

\[
\begin{array}{cccccc}
\text{Coefficients} & \text{Standard Error} & \text{t Stat} & \text{P-value} & \text{Lower 95%} & \text{Upper 95%} \\
\hline
\text{Intercept} & 34.344 & 1.482 & 23.17 & 7.493E-15 & 31.230 & 37.458 \\
\text{Ln Speed} & -5.186 & 0.2326 & -22.29 & 1.467E-14 & -5.674 & -4.697 \\
\hline
\end{array}
\]
(i) For $y = \ln \text{Life}$, find the estimated variance of $\hat{y}$ when speed = 400. 0.0109

$$\text{Var}(\hat{y}) = \text{Var}(\bar{y} + (x - \bar{x})b_1) = \text{Var}(\bar{y}) + \text{Var}((x - \bar{x})b_1) = \frac{\sigma^2}{n} + (x - \bar{x})^2 \text{Var}(b_1)$$

$$\text{Var}(\hat{y}) \approx \frac{0.06491}{20} + (\ln(400) - 6.3677)^2 * 0.0541$$

or

$$\text{Var}(\hat{y}) = \frac{\sigma^2}{n} + (x - \bar{x})^2 \text{SE}_{b_1}^2$$

$$\text{Var}(\hat{y}) \approx \frac{0.06491}{20} + (\ln(400) - 6.3677)^2 * 0.2326^2$$

(j) For $y = \ln \text{Life}$, find the estimated standard error of $\hat{y}$ when speed = 400. 0.104

$$\text{SE}_{\hat{y}} = \sqrt{\text{Var}(\hat{y})} = \sqrt{0.0109}$$

(k) Find the 95% confidence interval for the mean of $\ln \text{Life}$ when speed = 400. Confirm from the plot that the value looks about right.

$$3.053 \quad 3.494$$

$$3.272 \pm 2.101 * 0.104$$

(l) Find the 95% confidence interval for the mean $\text{Life}$ when speed = 400.

$$21.2 \quad 32.9$$

(m) For $y = \ln \text{Life}$ and speed = 400, find the estimated $\text{Var}(y_{\text{new}} - \hat{y})$. 0.0758

$$\text{Var}(y_{\text{new}} - \hat{y}) = \text{Var}(y_{\text{new}} - \hat{y}) = \text{Var}(y_{\text{new}}) + \text{Var}(\hat{y}) = \sigma^2 + \text{Var}(\hat{y})$$

$$\text{Var}(y_{\text{new}} - \hat{y}) \approx \text{MS Residual} + \text{Var}(\hat{y}) = 0.06491 + 0.0109$$

or

$$\text{MS Residual} + \text{SE}_{\hat{y}}^2$$

(n) For $y = \ln \text{Life}$ and speed = 400, find the estimated standard error of $y_{\text{new}} - \hat{y}$. 0.2753

$$\text{SE}_{y_{\text{new}} - \hat{y}} = \sqrt{\text{Var}(y_{\text{new}} - \hat{y})} = \sqrt{0.0758}$$

(o) Find the 95% prediction interval for the next measured value of $\ln \text{Life}$ when speed = 400. Confirm from the plot that the value looks about right.

$$2.69 \quad 3.85$$

$$3.272 \pm 2.101 * 0.2753$$

(p) Find the 95% confidence interval for the next measured value of $\text{Life}$ when speed = 400.

$$14.7 \quad 47.0$$

(q) What is the average change in $\ln \text{Life}$ if speed is increased by 25%? -1.157

$$b_0 + b_1 * \ln(1.25 * \text{Speed}) - b_0 + b_1 * \ln(\text{Speed}) = b_1 * \ln(1.25) = -5.186 * \ln(1.25)$$
What is the ratio of predicted tool lives at an initial speed and at a 25% higher speed?

\[
\frac{\text{Life at speed } 1.25 \times S_1}{\text{Life at speed } S_1}
\]

Differences in a log scale correspond to ratios in the original, back-transformed scale.

\[
e^{-1.157}
\]

What is the % change in predicted tool life if speed is increased by 25%?

\[
(1 - 0.314) \times 100\% = -68.6\%
\]

Write the fitted equation relating tool life to speed. Write the equation in simplest terms.

\[
e^{34.344 - 5.186 \ln(Speed)} = e^{34.344} e^{-5.186(Speed^{-5.186})} = 8.230 \times 10^{14} \times Speed^{-5.186}
\]

Use your equation in (t) to find estimated lives for speed = 400 and for a 25% increase in speed. Confirm your answers from the recorded data given above.

\[
\begin{align*}
\text{Speed} &= 400: 8.230 \times 10^{14} \times 400^{-5.186} = 26.370 \\
\text{Speed} &= 500: 8.230 \times 10^{14} \times 500^{-5.186} = 8.290
\end{align*}
\]

Use the values in part (u) to confirm your ratio in part (r).

\[
\frac{26.370}{8.290} = 0.318
\]

This is the same as in (r) except for round-off errors.

An approximate percent change of life when speed increases by 25 % would be \(-5.186 \times 25\% = -130\%\). Why isn't this a good approximation for the effect on tool life with a 25% increase in speed?

A change of 25%, for example from speed=400 to speed=500, is a big change and the fitted function curves a lot in this range of values. Using the tangent line from the derivative at speed = 400 does not approximate the function well.