### Variance Estimates and Degrees of Freedom

- For any t-test or confidence interval
  - $df = \text{degrees of freedom associated with estimate of } \sigma^2$
  - $\sigma^2$ is the variance of values measured under the same conditions
    - Within the same treatment group in chapter 6 t-tests or factorial designs
    - With the same $x$ values in regression.

<table>
<thead>
<tr>
<th>Estimate of $\sigma^2$</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^2$</td>
<td>$n - 1$</td>
</tr>
<tr>
<td>$s_{d}^2$</td>
<td>$n - 1$</td>
</tr>
</tbody>
</table>

#### Assuming Equal Variances

- **2 Independent Groups**
  \[ s^2_p = \frac{(n_1 - 1)s^2_1 + (n_2 - 1)s^2_2}{(n_1 - 1) + (n_2 - 1)} \]
  \[ (n_1 - 1) + (n_2 - 1) \]

- **Multiple Groups and Factorial**
  \[ s^2_p = \frac{\sum_{i=1}^{t} (n_i - 1) \cdot s^2_i}{\sum_{i=1}^{t} (n_i - 1) \cdot \sum (n_i - 1) } \]  
  \[ MS Error = MS Within = MS Residual \]

#### Regression with a Single X
  \[ MS Error = MS Residual \]
  \[ n - 2 \]

- **MS Error or MS Within or MS Residual** are just different names that get used for MS Residual
  - $MS$ Residual or $MS$ Error or $MS$ Within estimates the variance of residuals, errors, $y$ values with the same $x$ value in regression, or $y$ values within the same treatment group.

- For factorial data
  - The same estimate of $\sigma^2$ and the same $df$ are use for any confidence intervals and t-tests,
  - Regardless of which treatment groups or combinations of treatment groups we are comparing.
  - The Estimate and SE Estimate will change depending on the comparison in the confidence interval or t-tests,
  - But $MS Error$ will be used for $\sigma^2$ in calculating any standard errors.

- **Special case for * and **
  - With equal n's, the pooled variance is a straight average of all variances.
  - Two groups: $s^2_p = \frac{s^2_1 + s^2_2}{2}$
  - Multiple groups: $s^2_p = \frac{s^2_1 + s^2_2 + \ldots + s^2_t}{t}$ for $t$ treatments.